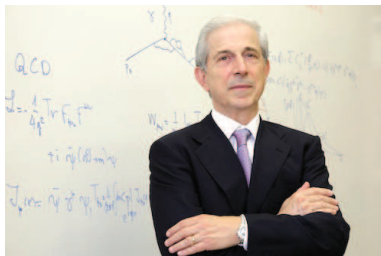


New horizons with Non-linear supersymmetry

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- Motivations
- Non-linear SUSY and the goldstino
- Non-linear SUSY and the MSSM
- Non-linear SUSY and Starobinsky inflation
- Extended supersymmetry and brane dynamics

Why study goldstino interactions:

- Effective field theory of SUSY breaking at low energies $m_\chi \ll m_{\text{susy}}$
e.g. gauge mediation dominant vs gravity mediation

χ : longitudinal gravitino with $m_\chi \simeq \frac{m_{\text{susy}}^2}{M_{\text{Planck}}} \lesssim m_{\text{soft}} \ll m_{\text{susy}}$

$M_{\text{Planck}} \rightarrow \infty$: SUGRA decoupled

massless χ coupled to matter $\sim 1/m_{\text{susy}}$

- Brane dynamics: half SUSY of the bulk broken but NL realized
 \Rightarrow strongly constrain coupling of brane to bulk fields

Non-linear SUSY transformations:

$$\delta\chi_\alpha = \frac{\xi_\alpha}{\kappa} + \kappa \Lambda_\xi^\mu \partial_\mu \chi_\alpha \quad \Lambda_\xi^\mu = -i(\chi\sigma^\mu\bar{\xi} - \xi\sigma^\mu\bar{\chi})$$

κ : goldstino decay constant (SUSY breaking scale) $\kappa = (\sqrt{2}m_{\text{susy}})^{-2}$

Volkov-Akulov action:

Define the 'vierbein': $E_\mu^a = \delta_\mu^a + \kappa^2 t_\mu^a \quad t_\mu^a = i\chi\overset{\leftrightarrow}{\partial}_\mu\sigma^a\bar{\chi}$

$\delta(\det E) = \kappa \partial_\mu (\Lambda_\xi^\mu \det E) \Rightarrow$ invariant action:

$$S_{VA} = -\frac{1}{2\kappa^2} \int d^4x \det E = -\frac{1}{2\kappa^2} - \frac{i}{2} \chi\sigma^\mu\overset{\leftrightarrow}{\partial}_\mu\bar{\chi} + \dots$$

D-brane examples

Type II (closed) strings on $4d$ Minkowski $M_4 \times X_6$ internal $6d$ manifold

X_6 flat $\Rightarrow N = 8$ SUSY ; X_6 Calabi-Yau $\Rightarrow N = 2$ SUSY

Single stack of N Dp -branes \Rightarrow half SUSY is spontaneously broken $p \geq 3$

$(p - 3)$ dims wrapped around cycles in $X_6 \Rightarrow 4d$ effective field theory

- Gauge group: $G = U(N)$ (generically)
- SUSY: half remains unbroken Q_e ; other half NL realized Q_o
broken SUSY commutes with $G \Rightarrow$ goldstino = $U(1)$ gaugino of Q_e

Intersecting branes: useful framework for model building

Standard Model embedding

Two D-brane stacks: $N_1 Dp_1$ and $N_2 Dp_2$

⇒ bifundamental matter on their intersections: chiral fermions

L-SUSY: generally broken but preserved for special intersection angles

e.g. for $X_6 = T^2 \times T^2 \times T^2$ when $\theta_1 + \theta_2 + \theta_3 = 0$

NL-SUSY: generally all (both $U(1)_1 \times U(1)_2$ gauginos = goldstinos)

special angles ⇒ only a linear combination

Remark: string consistency (e.g. tadpole cancellation) ⇒ need orientifolds

non-dynamical planes ⇒ break half-SUSY explicitly

⇒ goldstino gets a volume suppressed mass

NL-SUSY only locally → restored in the large volume limit

Constrained superfields

Rocek-Tseytlin '78, Lindstrom-Rocek '79, Komargodski-Seiberg '09

spontaneous global SUSY: no supercharge but still conserved supercurrent

⇒ superpartners exist in operator space (not as 1-particle states)

⇒ constrained superfields: 'eliminate' superpartners

Goldstino: chiral superfield X_{NL} satisfying $X_{NL}^2 = 0$ ⇒ [27]

$$\begin{aligned} X_{NL}(y) &= \frac{\chi^2}{2F} + \sqrt{2}\theta\chi + \theta^2 F & y^\mu &= x^\mu + i\theta\sigma^\mu\bar{\theta} \\ &= F\Theta^2 & \Theta &= \theta + \frac{\chi}{\sqrt{2}F} \end{aligned}$$

$$\mathcal{L}_{NL} = \int d^4\theta X_{NL}\bar{X}_{NL} - \frac{1}{\sqrt{2}\kappa} \left\{ \int d^2\theta X_{NL} + h.c. \right\} = \mathcal{L}_{Volkov-Akulov}$$

$$F = \frac{1}{\sqrt{2}\kappa} + \dots$$

Reduce the 'little' fine-tuning in MSSM

MSSM upper bound on the lightest scalar mass:

$$m_h^2 \lesssim m_Z^2 \cos^2 2\beta + \frac{3}{(4\pi)^2} \frac{m_t^4}{v^2} \left[\ln \frac{m_{\tilde{t}}^2}{m_t^2} + \frac{A_t^2}{m_{\tilde{t}}^2} \left(1 - \frac{A_t^2}{12m_{\tilde{t}}^2} \right) \right] \lesssim (130 \text{ GeV})^2$$

$$m_h \simeq 126 \text{ GeV} \Rightarrow m_{\tilde{t}} \simeq 3 \text{ TeV} \text{ or } A_t \simeq 3m_{\tilde{t}} \simeq 1.5 \text{ TeV}$$

\Rightarrow **compatible with SUSY** but % to a few ‰ fine-tuning

$$\text{minimum of the potential: } m_Z^2 = 2 \frac{m_1^1 - m_2^2 \tan^2 \beta}{\tan^2 \beta - 1} \sim -2m_2^2 + \dots$$

$$\begin{aligned} \text{RG evolution: } m_2^2 &= m_2^2(M_{\text{GUT}}) - \frac{3\lambda_t^2}{4\pi^2} m_{\tilde{t}}^2 \ln \frac{M_{\text{GUT}}}{m_{\tilde{t}}} + \dots \\ &\sim m_2^2(M_{\text{GUT}}) - \mathcal{O}(1)m_{\tilde{t}}^2 + \dots \end{aligned}$$

Goldstino couplings to matter supermultiplets

replace spurion superfield $S = m_{\text{soft}}\theta^2$ by goldstino constrained superfield

$$S \rightarrow \sqrt{2}k m_{\text{soft}} X_{NL} = \frac{m_{\text{soft}}}{m_{\text{susy}}} X_{NL}$$

⇒ Non-linear MSSM

F -auxiliary in X_{NL} : dynamical field with no derivatives to be solved

$$-\bar{F} = m_{\text{susy}}^2 + \frac{B_{\mu}}{m_{\text{susy}}^2} h_1 h_2 + \frac{A_u}{m_{\text{susy}}^2} \tilde{u}_R \tilde{q} h_2 + \dots$$

⇒ compact form for all goldstino couplings at linear and non-linear level

with

$$\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{X_{NL}} + \mathcal{L}_H + \mathcal{L}_m + \mathcal{L}_{AB} + \mathcal{L}_g$$

$$\mathcal{L}_H = \sum_{i=1,2} \frac{m_i^2}{m_{SUSY}^4} \int d^4\theta X_{NL}^\dagger X_{NL} H_i^\dagger e^{V_i} H_i$$

$$\mathcal{L}_m = \sum_{\Phi} \frac{m_{\Phi}^2}{m_{SUSY}^4} \int d^4\theta X_{NL}^\dagger X_{NL} \Phi^\dagger e^V \Phi \quad ; \quad \Phi = Q, U^c, D^c, L, E^c$$

$$\begin{aligned} \mathcal{L}_{AB} = & \frac{1}{m_{SUSY}^2} \int d^2\theta X_{nl} (A_u H_2 Q U^c + A_d Q D^c H_1 + A_e L E^c H_1) \\ & + \frac{B\mu}{m_{SUSY}^2} \int d^2\theta X_{NL} H_1 H_2 + h.c. \end{aligned}$$

$$\mathcal{L}_g = \sum_{i=1}^3 \frac{1}{8g_i^2} \frac{m_{\lambda_i}}{m_{SUSY}^2} \int d^2\theta X_{NL} \text{Tr} [W^\alpha W_\alpha]_i + h.c.$$

Higgs potential is modified:

$$V = V_{MSSM} + 2\kappa^2 |m_1^2| |h_1|^2 + m_2^2 |h_2|^2 + B\mu h_1 h_2 + \mathcal{O}(\kappa^4) \Rightarrow$$

$m_{1,2}, B\mu$: soft mass parameters, μ : higgsino mass

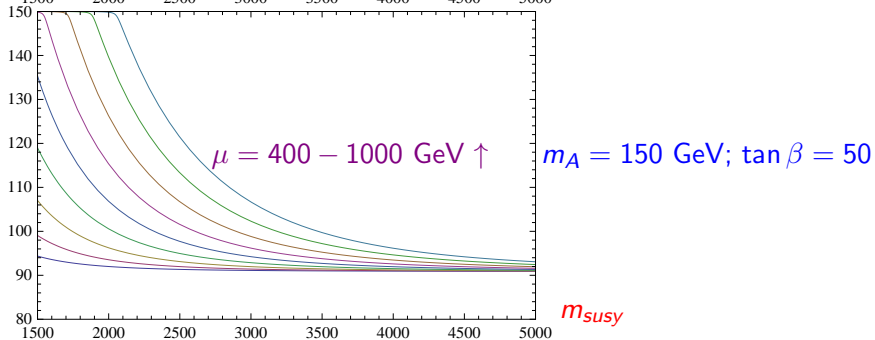
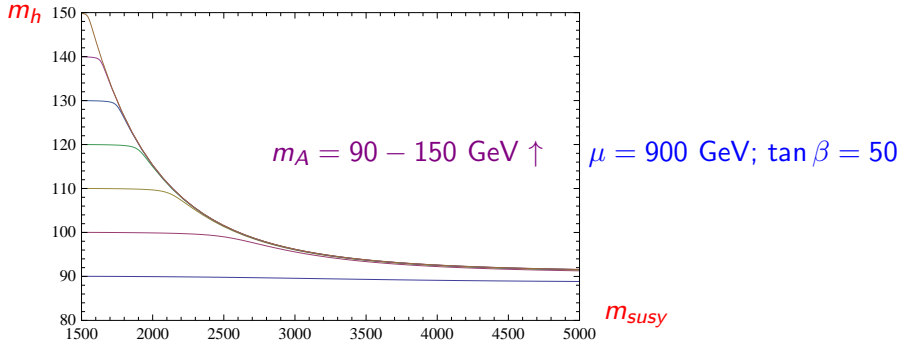
Classical value of light higgs mass can be increased significantly

for $m_{SUSY} \sim$ a few TeV

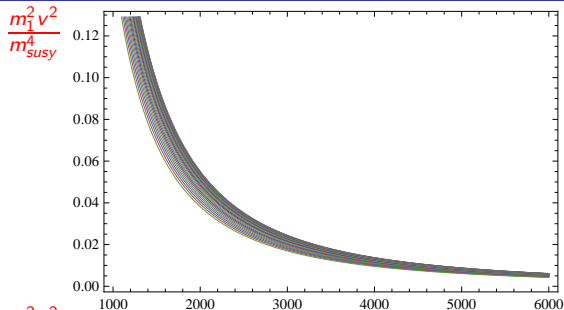
large $\tan \beta$ limit: $m_h^2 = m_Z^2 + \frac{v^2}{2m_{SUSY}^2} (2\mu^2 + m_Z^2)^2 + \dots$

Quartic higgs coupling increases for large soft masses \Rightarrow

MSSM 'little' fine tuning of the EW scale is alleviated

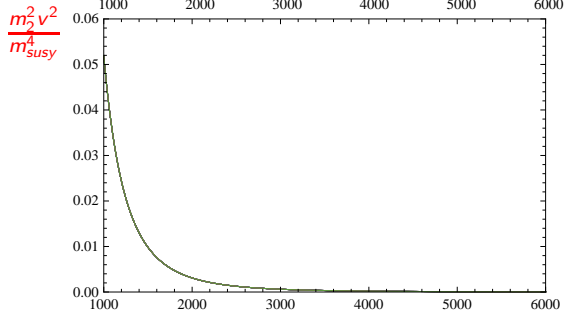


Validity of perturbative expansion: $m_i^2 v^2 / m_{\text{susy}}^4 \ll 1$



$$\mu = 900 \text{ GeV}; \tan \beta = 50$$

$$m_A = 90 - 650 \text{ GeV} \uparrow$$



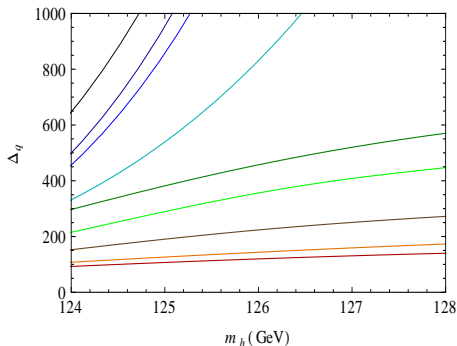
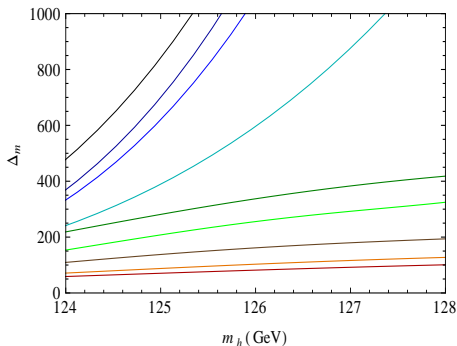
Fine-tuning measures:

Ellis-Enqvist-Nanopoulos-Zwirner '86
 Barbieri-Giudice '88, Anderson-Castano '94

$$\Delta_m = \max |\Delta_{\gamma^2}|, \quad \Delta_q = \left\{ \sum_{\gamma} \Delta_{\gamma^2}^2 \right\}^{1/2}, \quad \text{with } \Delta_{\gamma^2} \equiv \frac{\partial \ln v^2}{\partial \ln \gamma^2}$$

$$\gamma = \{m_0, m_{12}, A_t, B_0, \mu_0\}, \quad v = \text{EW scale}$$

The Constrained Non-Linear MSSM



Minimal values of Δ_m (left) and Δ_q (right) for $\tan\beta = 10$ and m_{SUSY} from the lowest to the top curve: 2.8 TeV (red), 3.2 TeV (orange), 3.9 TeV (brown), 5 TeV (green), 5.5 TeV (dark green), 6.3 TeV (cyan), 7.4 TeV (blue), 8 TeV (dark blue), 8.7 TeV (black).

$$m_{SUSY} \simeq 3 \text{ TeV} \Rightarrow \Delta^{NL} \sim \Delta/10 \text{ [20]}$$

Invisible decays of Higgs and Z boson

Other relevant couplings at order $\mathcal{O}(\kappa)$: $\frac{1}{m_{\text{SUSY}}^2} \times$

$$\left(m_1^2 \chi \psi_{h_1^0} h_1^{0*} + m_2^2 \chi \psi_{h_2^0} h_2^{0*} \right) + B\mu \left(\chi \psi_{h_2^0} h_1^0 + \chi \psi_{h_1^0} h_2^0 \right) + \sum_{i=1,2} \frac{m_{\lambda_i}}{\sqrt{2}} \tilde{D}_i^a \chi \lambda_i^a + \sum_{i=1}^2 \frac{m_{\lambda_i}}{\sqrt{2}} \chi \sigma^{\mu\nu} \lambda_i^a F_{\mu\nu, i}^a + h.c.$$

\Rightarrow invisible higgs decay $h \rightarrow \chi + \text{NLSP}$ if NLSP is light enough

otherwise inverse decay studied in the past

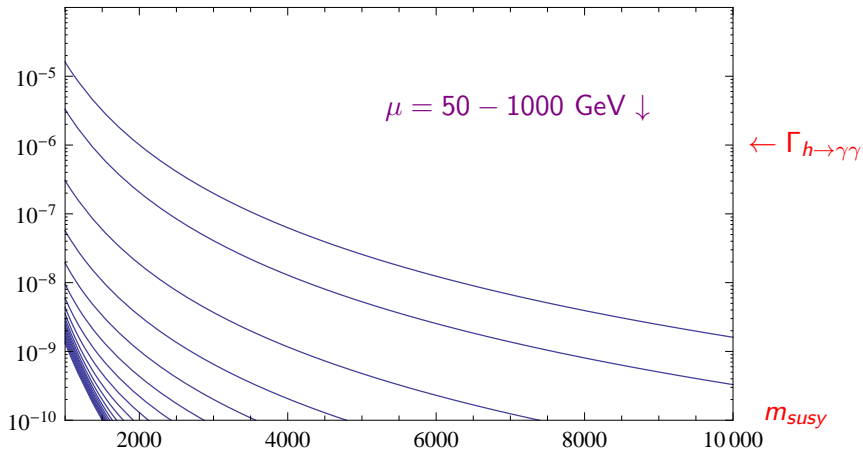
taking also into account the goldstino components of higgsinos/gauginos

from SUSY terms: $\sim h_i^0 \lambda \tilde{h}_i^0$

Similarly $Z \rightarrow \chi + \text{NLSP} \Rightarrow m_{\text{SUSY}} \gtrsim 400\text{-}700 \text{ GeV}$ from Z-width

$\Delta\Gamma_Z \lesssim 2.3 \text{ MeV}$

$\Gamma_{h \rightarrow \chi + \text{NLSP}}$ $m_A = 150 \text{ GeV}; \tan \beta = 50; (m_{\lambda_1}, m_{\lambda_2}) = (70, 150) \text{ GeV}$



$$K = -3 \log(1 - X\bar{X}) \equiv 3X\bar{X} \quad ; \quad W = fX + W_0 \quad \quad X \equiv X_{NL}$$

$$\Rightarrow \quad V = \frac{1}{3}|f|^2 - 3|W_0|^2 \quad ; \quad m_{3/2}^2 = |W_0|^2$$

- V can have any sign **contrary to global NL SUSY**
- NL SUSY in flat space $\Rightarrow f = 3 m_{3/2} M_p$
- Dual gravitational formulation: $\mathcal{R}^2 = 0$ **\leftarrow chiral curvature superfield**
- Minimal SUSY extension of R^2 gravity

Starobinsky model of inflation

$$\mathcal{L} = \frac{1}{2}R + \alpha R^2$$

$$\text{Lagrange multiplier } \phi \Rightarrow \mathcal{L} = \frac{1}{2}(1 + 2\phi)R - \frac{1}{4\alpha}\phi^2$$

Weyl rescaling \Rightarrow equivalent to a scalar field with exponential potential:

$$\mathcal{L} = \frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - \frac{M^2}{12} \left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2 \quad M^2 = \frac{3}{4\alpha}$$

Note that the two metrics are not the same

supersymmetric extension:

add D-term $\mathcal{R}\bar{\mathcal{R}}$ because F-term \mathcal{R}^2 does not contain R^2

\Rightarrow brings two chiral multiplets [23]

Planck XVI

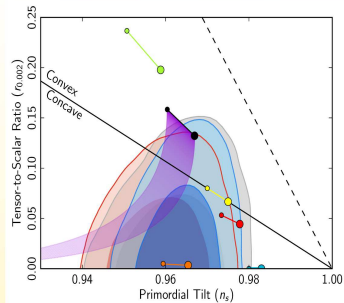
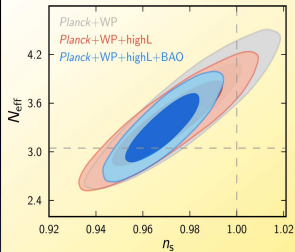
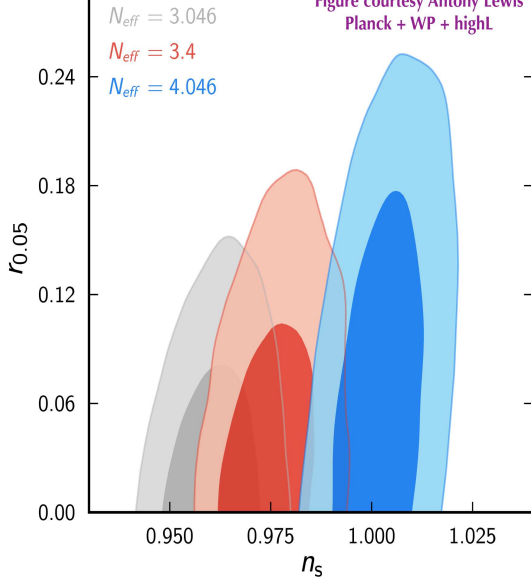


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

Figure courtesy Antony Lewis Planck + WP + highL



Chiral curvature superfield:

$$\mathcal{R} = \left(u, \gamma^{mn} \mathcal{D}_m \psi_n, -\frac{1}{2} R - \frac{1}{3} A_m^2 + i \mathcal{D}^m A_m - \frac{1}{3} u \bar{u} \right)$$

u, A_m : auxiliary fields of the 'old minimal' $N = 1$ SUGRA

$$\mathcal{L} = \mathcal{R} \bar{\mathcal{R}}|_D + W(\mathcal{R})|_F = C \bar{C}|_D + [\Lambda(C - \mathcal{R}) + W(C)]_F$$

C, Λ : auxiliary chiral multiplets

$W(C)$ can be set to constant W_0 by Λ redefinition + few manipulations

$\Rightarrow N = 1$ SUGRA with 2 chiral multiplets

SUSY extension of Starobinsky model

$$K = -3\ln(T + \bar{T} - C\bar{C}) \quad ; \quad W = MC(T - \frac{1}{2})$$

- T contains the inflaton: $\text{Re } T = e^{\sqrt{\frac{2}{3}}\phi}$
- $C \sim \mathcal{R}$ is unstable during inflation

⇒ add higher order terms to stabilize it

e.g. $C\bar{C} \rightarrow h(C, \bar{C}) = C\bar{C} - \zeta(C\bar{C})^2$ Kallos-Linde '13

- SUSY is broken during inflation with C the goldstino superfield

Minimal SUSY extension that evades stability problem:

replace C by the non-linear multiplet X

Non-linear Starobinsky supergravity

$$K = -3\ln(T + \bar{T} - X\bar{X}) \quad ; \quad W = MXT + fX + W_0 \quad \Rightarrow$$

$$\mathcal{L} = \frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - \frac{M^2}{12} \left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2 - \frac{1}{2}e^{-2\sqrt{\frac{2}{3}}\phi}(\partial a)^2 - \frac{M^2}{18}e^{-2\sqrt{\frac{2}{3}}\phi}a^2$$

- axion a much heavier than ϕ during inflation, decouples:

$$m_\phi = \frac{M}{3}e^{-\sqrt{\frac{2}{3}}\phi_0} \ll m_a = \frac{M}{3}$$

- inflation scale M independent from NL-SUSY breaking scale f

⇒ compatible with low energy SUSY

- string realization? [28]

Goldstino in multiplet of $N = 1$ SUSY: **vector or chiral?**

brane dynamics \Rightarrow Maxwell goldstino multiplet

gauge chiral multiplet $|_{N=2} \mathcal{W} = (\text{vector } W + \text{chiral } X)_{N=1}$

$$\mathcal{W}(y, \theta, \tilde{\theta}) = X(y, \theta) + i\sqrt{2}\tilde{\theta}W(y, \theta) - \tilde{\theta}^2 \left[\frac{1}{4} \overline{DDX}(y, \theta) + \frac{1}{2\kappa} \right]$$

allow partial SUSY breaking $N = 2 \rightarrow N = 1$

I.A.-Partouche-Taylor '96

$$\delta^* X = i\sqrt{2}\eta^\alpha W_\alpha \quad \delta^* W_\alpha = \frac{i}{\sqrt{2\kappa}}\eta_\alpha + \dots \leftarrow \text{linear SUSY}$$

$$\mathcal{L}_{Maxwell}^{N=2} = -\frac{1}{8} \int d^2\theta d^2\tilde{\theta} \mathcal{W}^2 + h.c. = \int d^2\theta \left[\frac{1}{2} W^2 - \frac{1}{4} X \overline{DDX} - \frac{1}{2\kappa} X \right] + h.c.$$

DBI action

Non-linear $N = 2$ constraint: $\mathcal{W}_{NL}^2 = 0$

$$\Rightarrow X^2 = 0 \quad , \quad XW_\alpha = 0 \quad , \quad WW - \frac{1}{2}X\overline{DDX} = \frac{1}{\kappa}X \quad [9]$$

$$X = \kappa W^2 - \kappa^3 \bar{D}^2 \frac{W^2 \overline{W}^2}{1+A_+ + \sqrt{1+2A_+ + A_-^2}} \quad A_\pm = \frac{\kappa^2}{2} \left(D^2 W^2 \pm \bar{D}^2 \overline{W}^2 \right) = \pm A_\pm^*$$

$$\Rightarrow \mathcal{L}_{NL}^{N=2} = \frac{1}{4\kappa} \int d^2\theta X + h.c.$$

$$= \frac{1}{8\kappa^2} \left(1 - \sqrt{-\det(\eta_{\mu\nu} + 2\sqrt{2}\kappa F_{\mu\nu})} \right) + \dots = \mathcal{L}_{\text{DBI}} \leftarrow \text{D-brane}$$

The FI-term is also invariant under NL SUSY

$$\mathcal{L}_{FI} = \xi \int d^4\theta V; \quad W = -\frac{1}{4}\bar{D}^2 DV; \quad \delta^* V = \frac{i}{2\kappa} (\eta D + \bar{\eta} \bar{D}) \theta^2 \bar{\theta}^2 + \dots$$

$$\Rightarrow \mathcal{L}_{\text{Max}}^{NL} = \mathcal{L}_{NL}^{N=2} + \mathcal{L}_{FI}$$

Conclusions

Non-linear supersymmetry: powerful tool for studying:

- low energy SUSY breaking $E \ll m_{\text{susy}} \sim 1/\sqrt{\kappa}$
Volkov-Akulov action and goldstino χ couplings to matter
- $m_{\text{soft}} \lesssim E \ll m_{\text{susy}}$: goldstino \equiv spurion coupled to supermultiplets
→ Non-linear MSSM : narrow but interesting region
 - new quartic higgs coupling \Rightarrow can increase the higgs mass
reduce the MSSM fine tuning of the EW scale
- coupling to supergravity: straightforward but several open questions
minimal SUGRA extension of Starobinsky model for inflation
- brane effective actions \Rightarrow brane dynamics
 $N = 2$ NL SUSY \Rightarrow DBI action and couplings to bulk fields