FloratosFest2014

New Horizons in Particles, Strings & Membranes

Athens, Oct. 10, 2014

WELCOME TO THE CLUB MANΩΛH

GAUGE THEORIES
AND
NON-COMMUTATIVE GEOMETRY

FF2014

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- 4) Large N gauge theories and matrix models.

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- 4) Large N gauge theories and matrix models.
- 5) The construction of gauge theories using the techniques of non-commutative geometry.

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- Define a * product

  $f * g = e^{\frac{i}{2}\frac{\partial}{x_\mu}\theta_{\mu\nu}\frac{\partial}{y_\nu}} f(x)g(y)|_{x=y}$

All computations can be viewed as expansions in θ

expansions in the external field

More efficient ways?

Large N field theories

- $\phi^i(x)\ i = 1, ..., N\ ;\ N \to \infty$

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- For a Yang-Mills theory, the resulting expression is local

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- with $\qquad [z_1, z_2] = \frac{2i}{N}$

$$[A_\mu(x), A_\nu(x)] \to \{\mathcal{A}_\mu(x, z_1, z_2), \mathcal{A}_\nu(x, z_1, z_2)\}_{Moyal}$$

$$[A_\mu(x), \Omega(x)] \to \{\mathcal{A}_\mu(x, z_1, z_2), \Omega(x, z_1, z_2)\}_{Moyal}$$

$$\int d^4x \;\; Tr\,(F_{\mu\nu}(x)F^{\mu\nu}(x)) \quad \to$$
$$\int d^4x dz_1 dz_2 \; \mathcal{F}_{\mu\nu}(x, z_1, z_2) * \mathcal{F}^{\mu\nu}(x, z_1, z_2)$$

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- Question: Is there a space on which Internal symmetry transformations act as Diffeomorphisms?
- Answer: Yes, but it is a space with non-commutative geometry. A space defined by an algebra of matrix-valued functions

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- *New predictions for the B.E.H. mass?*

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- Related question: Is there a B.R.S. symmetry for this model?

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- It will depend on our ability to simplify the mathematics sufficiently, or to master them deeply, in order to get new insights
- We need somebody with knowledge and imagination

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