Double Magic to E. Floratos

Athens October 10, 2014
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Square-Line-Triangles-Square

- (Hurwitz 1898…Column of real composition algebras)
- Freudenthal 1950's Tits 1960's
- Manin cubic forms 1972
- Julia 1980 and +al.1999 / Pure SUGRAS 4D
- Cvitanovic (1977) 1979-2003, Deligne et al. (2002)
- Gunaydin, Sierra, Townsend 1983. N=2 SUGRA+Vector Multiplets (Special geometries)

Real division algebras
First magic (R),C,H / O

	$n=\dim_R(A)$
A=R	1
C	2
H	4
O	8

Freudenthal, Rosenfeld, Tits Magic square(s): « Ms60's »

Octonions	H	C	R	compact forms
$F_4(-52)$	$C_3(-21)$	$A_2(-8)$	$A_1(-3)$	R
$E_6(-78)$	$A_5(-35)$	$A_2.A_2(-16)$	$A_2(-8)$	C
$E_7(-133)$	$D_6(-66)$	$A_5(-35)$	$C_3(-21)$	H
$E_8(-248)$	$E_7(-133)$	$E_6(-78)$	$F_4(-52)$	Octonions

Δ symmetry is under the exchange of division algebras A and B

Properties of composition algebras

	$n=\dim_R$	$Aut(J_3(A))$	$Aut(nT)$	$Aut(A)$
A=R	1	$SO(3)=A_1$	Z_2^2	e
C	2	$SU(3)=A_2$	$U_1^2.Z_2$	Z_2
H	4	$USp(3)=C_3$	A_1^3/Z_2	SO(3)
O	8	F_4	D4=Spin8	G_2

MSq. constructions. The Δ symmetry issue.

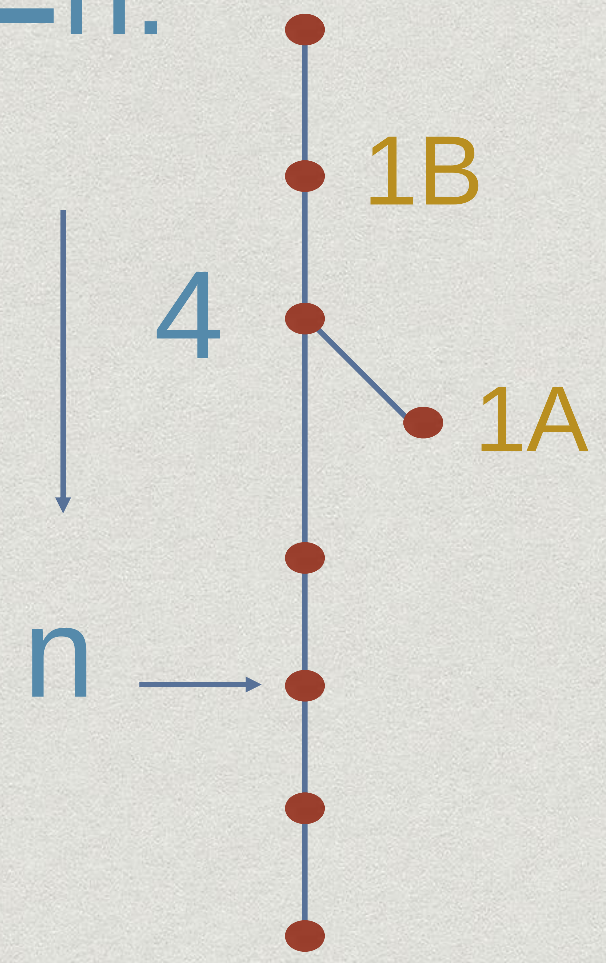
- Tits' first asymmetric construction:
- $\mathrm{Magic}G_{AB}=\mathrm{Der}A+\mathrm{Der}J^B+A'\mathrm{x}J'^B$
- Vinberg's manifestly Δ symmetric construction
- $\mathrm{Magic}G_{AB}=\mathrm{Tri}A+\mathrm{Tri}B+3\ A\mathrm{x}B$
- Trialities: Adams… see 0910.1828.

The 10 E_n's

$E_0=e$
$E_{1A}=R$ / $E_{1B}=A_1$
$E_2=R.A_1$
$E_3=A_1 x A_2$
$E_4=A_4$

$E_5=D_5$
E_6
E_7 : $E_{7(7)}$ D=4 $KE_7=SU(8)$
E_8

E_n:



The E_n Family (n=0,1A,1B, 2-8)
Algebraic geometry

- Complex rational surfaces (4 real dimensions) of Del Pezzo (CP2 with generic n=0 to 8 points blown up or CP1xCP1 (E_{1B})) have middle cohomology $H^2(M_4,Z)$ related to the root lattice of E_n (Manin 1972, Deligne?, $E_{1B}=A_1$)
- E_n grows with n along the «gravity line» (BJ 1980 Mt80) It corresponds to GL(11-D). Same for N=6,5,4 worked out then. In 1982 Tits-Satake diagrams: the maximal split subalgebra can be analysed with roots not G itself.

Cvitanovic Trapezoid: mT77

								Primitive invariants
F_4(-52)	C_3(-21)	A_2(-8)	A_1(-3)					S2,S3
E_6(-78)	A_5(-35)	A_2^2(-16)	A_2(-8)	U1^2				V^2,S3
E_7(-133)	D_6(-66)	A_5(-35)	C_3(-21)	A_1^3(-9)	A_1(-3)	U1	←	A2,S4 S2,A3
E_8(-248)	E_7(-133)	E_6(-78)	F_4(-52)	D_4(-28)	G_2(-14)	A_2(-8)	B_1(-3)	A_1(-3)

Faulkner-Ferrar 1971-77

Cvitanovic 1977-79 — 03

Magic triangle 1980 B.Julia (Pure + viable in 4d)

«Mt80» All factors simple=ADE or R.

	N_4=8	6	5	4	3	2	1	0
	N_2=16	12	10	8	6	4	2	0
D=11	Mgrav.							
10	R (A_1?)							
9	A_1.R							
8	A_2.A_1							
7	$A_{4(4)}$							
6	$D_{5(5)}$	$D_{3(-5)}$.A_1						
5	$E_{6(6)}$	$A_{5(-7)}$						
4	$E_{7(7)}$	$D_{6(-6)}$	$A_{5(-15)}$	D_3.$A_{1(1)}$	A_2.U(1)	A_1.U(1)	U(1)	Egrav.
3	$E_{8(8)}$	$E_{7(-5)}$	$E_{6(-14)}$	$D_{5(-13)}$	$A_{4(-8)}$	$A_{2(0)}$.A_1	$A_{1(1)}$.U(1)	A_1 /U(1)
2	E_9?	E_7^?	E_6^?	D_5^?				Geroch

D=6, N=6, D'A.F.K. '9711048

MT80

- D=3, 4, 6,10,and 11 double magic
- Partial Δ symmetry
- « Exceptional» symmetric spaces
- Compare D=3,4,6,10 single magic, BSS-KT Pure super Yang-Mills maximal dimensions N_3=1,2,4,8

Split Magic triangle 1999 Cremmer, BJ, Lu, Pope Mt99 IS Non-SUPERSYMMETRIC Δ-SYMMETRIC

	16	12	10	8	6	4	2	0	-2
D=11	e								SAY
10	R (A_1)	e							posterior
9	A_1.R	R							
8	$A_2.A_1$	A_1.R	A_1	?					
7	A_4	A_2.R	A_1.R	R	e				
6	D_5	$D_3.A_1$	A_1^2.R	R^2	R	?			
5	E_6	A_5	$A_2.A_2$	A_1^2.R	A_1.R	A_1			
4	E_7	D_6	A_5	$D_3.A_1$	A_2.R	A_1.R	R	e	
3	E_8	E_7	E_6	D_5	A_4	$A_2.A_1$	A_1.R	A_1	e

Two different double Magics

- The SUGRA magic (algebraic geometry?):
  Del Pezzo-CJ En’s E5=D5,E4=A4, E3=A2xA1

- The Universal magic (Invariant theory):
  Cvitanovic E’n’s E’5=F4, E’4=D4, E’3=G2

B.Gross-Deligne/Cvitanovic Magic triangle: Mt02

11	e								E_8
10	A_1								E_7
9	A_2	C^*							E_6
8	G_2	A_1	μ_3						F_4
7	D_4	A_1^3	C^{*2}	μ_2^2					D_4
6	F_4	C_3	A_2	A_1	μ_2^2				G_2
5	E_6	A_5	A_2^2	A_2	C^{*2}	μ_3			A_2
4	E_7	D_6	A_5	C_3	A_1^3	A_1	C^*		A_1
3	E_8	E_7	E_6	F_4	D_4	G_2	A_2	A_1	e

Δ-Symmetry manifest

- In E_8, E'_i and $E'_{(8-i)}$ centralize each other (E'_0=e, E'_4=D4…)
- Here Gij is the intersection of the centralizer of E'i with the centralizer of E'j in E8.
- CRAS 335 (2002) p877 B Gross and P Deligne
- What is not completely clear is the logic for the choice of groups and from the point of view of invariant theory the choice of adjoint representation's invariants needed.
- See below Vogel's universal Lie algebra inspired by Deligne 1999 unpublished and 2010

One more square/rectangle

- While visiting us at ENS Paris Günaydin, Sierra and Townsend discovered a full 4x4 magic square of dualities (1983). They considered N_4=2 supergravities with only extra vector multiplets: it is certainly not maximal magic, it is special magic (a second almost-square double magic).
- Magic has higher dimensional avatars.
- $2^{(7-D)}$, n_T-1…

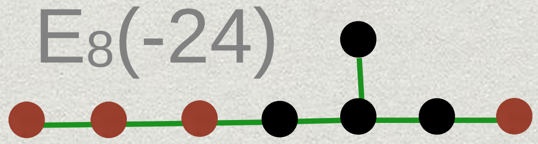
N_4=2 Magic square GST 1983

n=4(4+3.2^{k-1}) degrees of freedom «Ms83»+?

	n=112 n_T-1= 8	64 4	40 2	28 1
D=6 Rrk=1	D_5(-27)	D_3(-5).A_1(-3)	D_2(0).U1(-1)	A_1(1)
D=5 Rrk=0	F_4	C_3	A_2	A_1
D=5 Rrk=2	E_6(-26)	A_5(-7)	A_2^C(0)	A_2(2)
D=4 Rrk=3	E_7(-25)	D_6(-6)	A_5(1)	C_3(3)
D=3 Rrk=4	E_8(-24)	E_7(-5)	E_6(2)	F_4(4)

Geometric type

E_8(-24)



Imperial Sweep

The sequence 1,2,4,8, dimensions of R division

algebras, characterizes the space-time dimension of

pure SuperYang-Mills but appears also in 3D as the number

of rigid supersymmetries.

N_4=1,2,4=3 in 4D corresponds to 2,4,8 in 3D, one can add

N_3=1 and no more. Inspired by string left and right movers one

is led to SUGRA as double SUSY see 1309.0546 and
1301.4176

Binary sweet

- Question:

Which are the integers not bigger than 16

that can be written with at most two 1's in

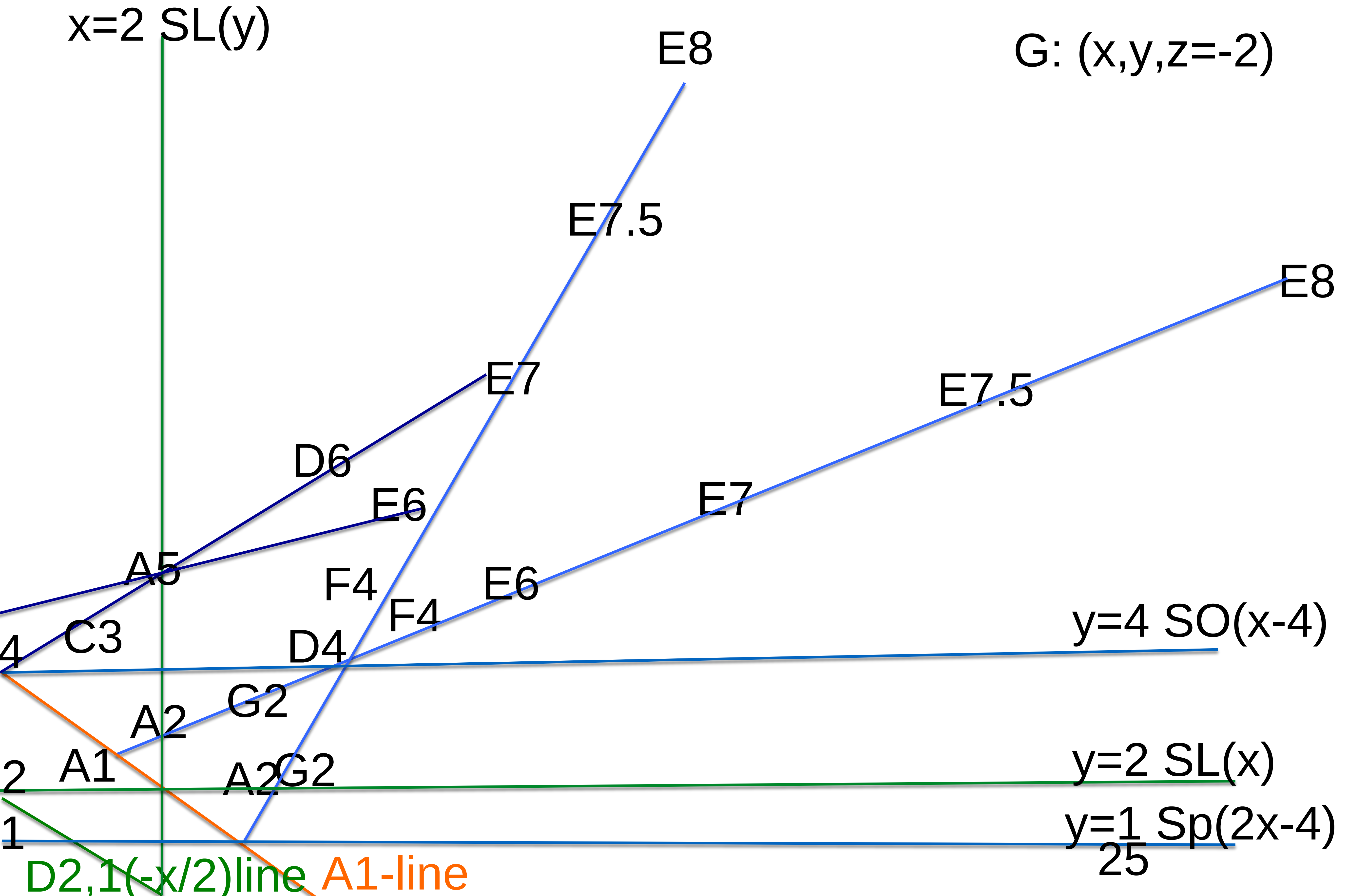
binary notation?

- (0,1) 2,3,4,5,6,8,9,10,12 and 16. (Not 14)
- Find the question 1,2,4,8 is the answer to!

Answer Binary sweep and gaps: 1,7,11,13,14,15

	1000	100	10	1=NL
NR=1				2
10			4	3
100		8	6	5
1000	16	12	10	9

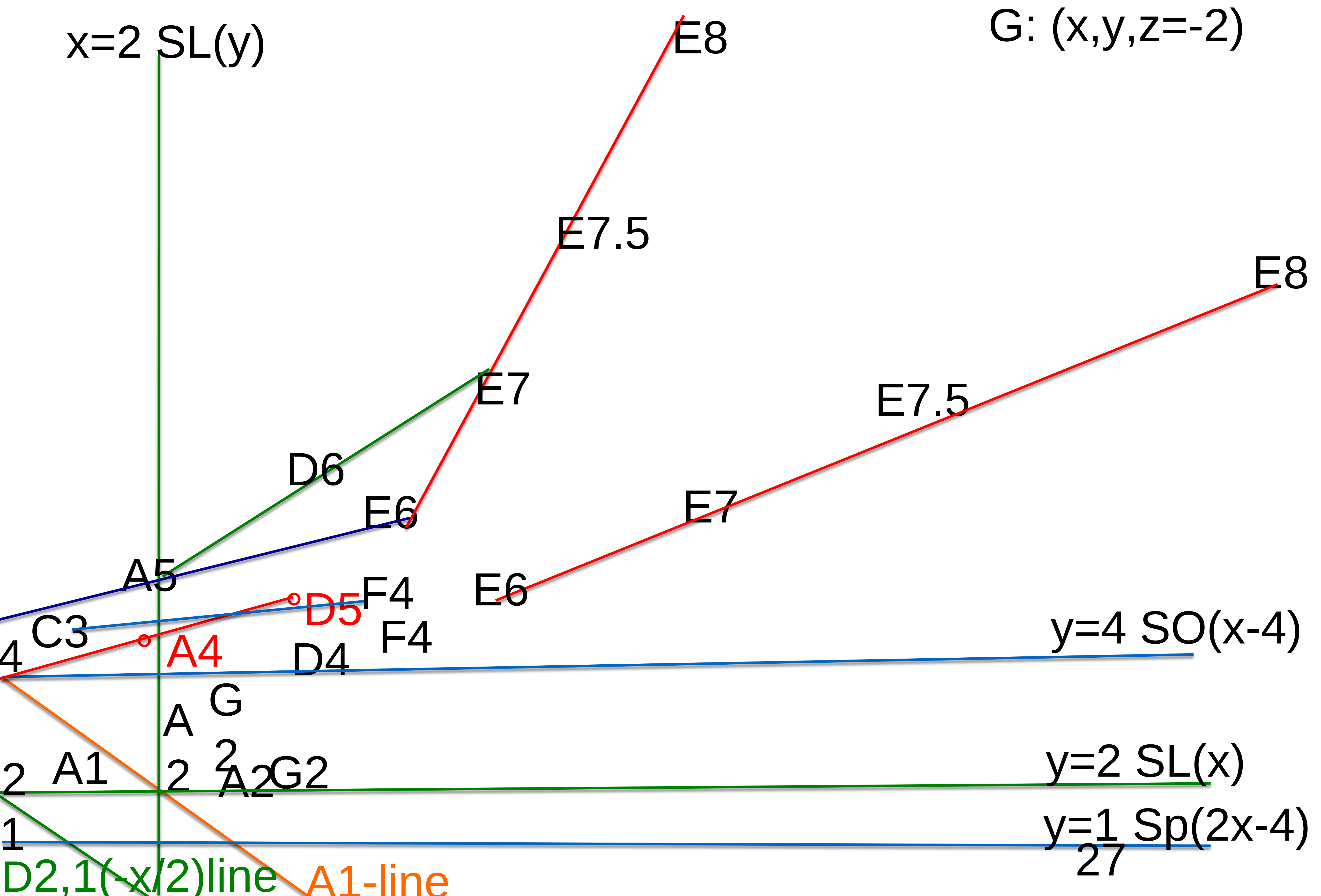
P. Vogel's $RP_3 / \sum_3$



B.Gross-Deligne/Cvitanovic Magic triangle: Mt02

11	e								E_8
10	A_1								E_7
9	A_2	C^*							E_6
8	G_2	A_1	μ_3						F_4
7	D_4	A_1^3	C^{*2}	μ_2^2					D_4
6	F_4	C_3	A_2	A_1	μ_2^2				G_2
5	E_6	A_5	A_2^2	A_2	C^{*2}	μ_3			A_2
4	E_7	D_6	A_5	C_3	A_1^3	A_1	C^*		A_1
3	E_8	E_7	E_6	F_4	D_4	G_2	A_2	A_1	e

MT 1980



G: (x,y,z=-2)
x=2 SL(y)
E8
E7.5
E8
E7
E7.5
D6
E6
E7
A5
F4
E6
D5
C3
F4
4
A4
D4
y=4 SO(x-4)
G
A
2
A1
2
A2
G2
2
y=2 SL(x)
1
y=1 Sp(2x-4)
D2,1(-x/2)line
A1-line

Split Magic triangle 1999 Cremmer, BJ, Lu, Pope

Mt99 IS Non-SUPERSYMMETRIC Δ-SYMMETRIC

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D=11	e								SAY
10	R (A_1)	e							posterior
9	A_1.R	R							
8	$A_2.A_1$	A_1.R	A_1	?					
7	A_4	A_2.R	A_1.R	R	e				
6	D_5	$D_3.A_1$	A_1^2.R	R^2	R	?			
5	E_6	A_5	$A_2.A_2$	A_1^2.R	A_1.R	A_1			
4	E_7	D_6	A_5	$D_3.A_1$	A_2.R	A_1.R	R	e	
3	E_8	E_7	E_6	D_5	A_4	$A_2.A_1$	A_1.R	A_1	e

Wish

- We did not coauthor any paper yet but here are some ideas:
- Projective Vogel space/ Membranes
- …

Work in progress

- Δ symmetry of Mt80, Mt99 and of Ms83 remains mysterious. Simpler definitions of the tables may make them obvious.

- Invent Half singularities.

E_n for n= 5.5, 3.5, 2.5, (1.5)

- $E'_{7.5}$ for N_3=14?

- Invent Half odd integer dimensions of space.

D=5.5, 7.5, 8.5, (9.5)

Binary sweet

(0,1) 2,3,4,5,6,8,9,10,12 and 16. (Not 14)

Thank You

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The quantum finiteness issue

- In July 1982 Michael Green bet (ask the referee Mark Grisaru for the all important details) that: « String theory but not SUGRA is a 4d finite quantum theory » against Bernard Julia who bet « that maximal SUGRA (D=4) could be finitely defined at the quantum level ».
- They had in mind the perturbative divergences issue, but Sine-Gordon-Thirring equivalence dates back to 1977. See Julia-Zee Phys. Rev. D 11, 2227–2232 (1975) for fermionization.