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$\mathcal{R}^2$ inflation from scale invariant supergravity and anomaly free superstrings with fluxes

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1. Introduction

This work is motivated by the observation that:
The pure $\mathcal{R}^2$ gravity is the only scale invariant gravity theory without ghosts.

It is conformally equivalent to a conventional Einstein gravity theory
with an extra scalar degree of freedom ϕ

$$S = \int d^4x \sqrt{|g|} \frac{1}{2} \left(\xi \, \mathcal{R} + \frac{\mathcal{R}^2}{8\mu^2} \right) \,, \qquad \text{the } \xi\text{−term violates the scale invariance}$$

Introducing a Lagrange multiplier t, one can replace the R^2 term with:

$$S_J = \int d^4x \sqrt{|g|} \left\{ \frac{1}{2} (\xi + 2t) \, \mathcal{R} - 4\mu^2 \, t^2 \right\}$$

t looks like a non-propagating field. This however is an illusion of the Jordan frame.

Performing a rescaling of the metric in order to obtain a canonical Einstein term:

$$g_{\mu\nu} \to g_{\mu\nu}\, e^{-\log(\xi+2t)}\,, \qquad \alpha\phi = \log(\xi + 2t), \qquad \alpha = \sqrt{\frac{2}{3}}$$

the action takes the following form in terms of the no-scale modulus ϕ:

$$S_E = \int d^4x \sqrt{|g|}\, \left[\frac{1}{2}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \mu^2\left(1 - \xi e^{-\alpha\phi}\right)^2\right]$$

$$V = \mu^2\left(1 - \xi e^{-\alpha\phi}\right)^2, \qquad \text{(Starobinsky inflationary potential)}$$

$\xi\mathcal{R} + \mathcal{R}^2$ theory was proposed by Starobinsky to describe inflationary cosmology. In order to create a successful density perturbation, $\delta\rho/\rho \sim 10^{-5}$, the mass scale μ must be as low as $\mu \sim 10^{-5}\, M \sim 10^{13} GeV$.

$$V(\phi \gg 0) \to \mu^2, \qquad V(\xi e^{-\alpha\phi_0} = 1) = 0, \qquad V(\phi \ll 0) \sim \mu^2\, \xi^2 e^{-2\alpha\phi}$$

For us the initial theory is assumed to be :
Scale invariant gravity $\mathcal{R}^2$ coupled to scale invariant matter.
The scale violating terms like the undressed $\mathcal{R}$-term proportional to ξ,
the bosonic mass terms and trilinear couplings, fermion masses,...are absent classically.

The quantum corrections associated to the matter interactions violate scale invariance inducing non-trivial scales due to renormalization effects, namely :

– Coleman-Weinberg mass terms via the quantum effective potential,
– Anomalous dimensions in the fields and couplings,
– Quantum anomalies of local interactions,

which break in general classical scale invariance.

In a fundamental theory, like superstring theory,
gravity and matter interactions make sense at the quantum level.
The quantum scale violating effects are controllable giving rise to quantitative results
$\longrightarrow$ The classical scale symmetry will appear softly broken by quantum mass-terms.

2. The $SO(1,1+n)$ generalization of the pure $\mathcal{R}^2$ theory

The first minimal non-susy extension of the $\mathcal{R}^2$ theory is achieved by introducing a conformally coupled matter.

$$S = \int d^4x\sqrt{|g|}\frac{1}{2}\left(\frac{\mathcal{R}^2}{8\mu^2} - \frac{1}{6}\Phi_i^2\,\mathcal{R} - \partial_\mu\Phi_i\partial^\mu\Phi_i - 2V_c(\Phi_i) + \ldots\right) ,$$

The fermionic and gauge boson parts of the action are invariant under conformal transformations. These parts do not affect the vacuum structure of the theory.

The linear $\mathcal{R}$ term is dressed by the fields Φ_i with conformal weight $w_\Phi = 1$. Introducing the Lagrange multiplier field t, the action in Jordan frame becomes linear in $\mathcal{R}$ dressed by t and Φ_i^2 both with weight $w = 2$:

$$S_J = \int d^4x\sqrt{|g|}\frac{1}{2}\left\{\left(2t - \frac{1}{6}\Phi_i^2\right)\mathcal{R} - \partial_\mu\Phi_i\partial^\mu\Phi_i - 2V_c(\Phi_i) - 8\mu^2\,t^2\right\} + \ldots$$

Performing a conformal rescaling of the metric

$$g_{\mu\nu} \to g_{\mu\nu}\, e^{-\log(2t - \frac{1}{6}\Phi_i^2)}\,, \quad \text{with} \quad \alpha\phi = \log\left(2t - \frac{1}{6}\Phi_i^2\right) \;\text{ and }\; \alpha = \sqrt{\frac{2}{3}}$$

The action in the Einstein frame:

$$S_E = \int d^4x \sqrt{|g|} \times$$

$$\frac{1}{2}\left[\mathcal{R} - \partial_\mu\phi\,\partial^\mu\phi - e^{-\alpha\phi}\,\partial_\mu\Phi_i\partial^\mu\Phi_i - 2e^{-2\alpha\phi}\,V_c(\Phi_i) - 2\mu^2\left(1 + \frac{e^{-\alpha\phi}\,\Phi_i^2}{6}\right)^2\,\right]$$

- The full classical action is invariant under a scale symmetry:

$$\alpha\phi \to \alpha\phi + 2\sigma\,, \qquad \Phi_i \to e^\sigma\Phi_i\,, \qquad g_{\mu\nu} \to g_{\mu\nu}\,,$$

provided the potential V_c is quartic: $V_c = \lambda_{ijkl}\,\Phi_i\Phi_j\Phi_k\Phi_l$.

The μ^2 term contains a positive cosmological constant term plus Φ^2 mass and Φ^4 interaction terms, dressed by powers of $e^{-\alpha\phi}$ in a scale invariant way.

- The scale symmetry is extended to the fermionic and gauge interacting parts of the theory as a remnant of the initial conformal invariance of the matter sector.
- The scalar manifold is a maximally symmetric space:

$$\mathcal{M}(\phi, \Phi_i) = \mathcal{H}^{n+1} \equiv \frac{SO(1,1+n)}{SO(1+n)}$$

- The vacuum solution is a de Sitter space-time with cosmological constant μ^2.

$$\phi = \text{const.} \qquad \Phi_i = 0 \qquad \text{with} \qquad m_i^2 = \tfrac{2}{3}\mu^2 \qquad (dS - induced\ mass)$$

- The explicit ϕ-dependence of the potential can be always absorbed into canonically normalized, scale invariant fields :

$$\hat{\Phi}_i = e^{-\frac{\alpha}{2}\phi}\,\Phi_i$$

In terms of $\hat{\Phi}_i$, the field ϕ appears only through its space-time derivatives.
ϕ remains always a flat direction when the scale symmetry is preserved.

- In the presence of scale violating term like $\xi\mathcal{R}$ introduces an explicit ϕ-dependence

$$V = V_c(\hat{\Phi}_i) + \mu^2 \left(1 - \xi e^{-\alpha\phi} + \frac{\hat{\Phi}_i^2}{6}\right)^2$$

The vacuum structure changes drastically. The scale violating term $\{-\xi e^{-\alpha\phi}\}$ induces a slow inflationary transition from the scale invariant de Sitter phase with $< \mathcal{R} >= 4\mu^2$ to the flat phase with $< \mathcal{R} >= 0$.

- The off-shell potential has three characteristic phases:

(i) Approximate de Sitter phase with negligible scale breaking term $\xi e^{-\alpha\phi} \ll 1$.
In this phase $\hat{\Phi}_i = 0$ thanks to the induced de Sitter mass $m_i^2 = \frac{2}{3}\mu^2$.

(ii) Flat-Minkowski $O(n)$-symmetric vacuum $\hat{\Phi}_i = 0$ with $\xi e^{-\alpha\phi} \sim 1$.

(iii) Scale-breaking dominant phase with $\xi e^{-\alpha\phi} \gg 1$, and $\hat{\Phi}_i^2 \neq 0$.

EITHER (a) V is growing exponentially, $V \sim \xi e^{-2\alpha\phi}$
like in the minimal Starobinsky model (non-degenerate V_c)

OR (b) V remains zero $V \equiv 0$ with $\hat{\Phi}_i^2/6 = (\xi e^{-\alpha\phi} - 1)$ (degenerate V_c)

The classical degeneracy with $V \equiv 0$ will be lifted at the quantum level by the induced scale violating terms coming from the quantum effective potential. These effects are more or less controllable around an approximate Minkowski background especially in supersymmetric theories.

- To understand the quantum origin of the gravitational scale breaking term $\xi\mathcal{R}$ it is necessary to move in the framework of a more fundamental theory, like for instance, suprergravity and superstring theories.

3. The minimal scale invariant $SU(1,1+n)$ supergravity

The minimal supersymmetric extension of the $SO(1,1+n)$ scale invariant theory is achieved by introducing the supersymmetric scalar partners of t and Φ_i which appear in the $SO(1,1+n)$ non-susy theory. Namely: the complex field T and the complex fields z^i, $i=1,...n$

$$T = t + ib \qquad z^i = |z^i| e^{i\theta^i}, \quad \text{with} \quad |z^i|^2 = \frac{\Phi_i^2}{6}, \qquad (\psi^I = \{T, z^i\})$$

$$S_J = \int d^4x \sqrt{|g|} \left[\frac{1}{2} Y(\psi^I) \left(\mathcal{R} + \frac{2}{3} A_\mu A^\mu \right) - J_\mu A^\mu + 3 Y_{I\bar{J}} \, \partial_\mu \psi^I \partial^\mu \bar{\psi}^{\bar{J}} - V_c \right],$$

$$Y(\psi^I) = T + \bar{T} - |z^i|^2, \quad Y_I = \frac{\partial Y}{\partial \psi^I}, \quad Y_{\bar{I}} = \frac{\partial Y}{\partial \bar{\psi}^{\bar{I}}}, \quad Y_{I\bar{J}} = \frac{\partial Y}{\partial \psi^I \partial \bar{\psi}^{\bar{J}}}.$$

*** We are using the "old minimal formalism for the supergravity multiplet."

We will not need to go beyond the "minimal" description at any point in this work !
The auxiliary axial vector field A_μ appears naturally together with $\mathcal{R}$.

$$A_\mu = \frac{3}{2}\frac{J_\mu}{Y} = \frac{3}{2}\, i\, \frac{Y_{\bar{I}}\partial_\mu \bar{\psi}^{\bar{I}} - Y_I \partial_\mu \psi^I}{Y}, \qquad \psi^I = \{T, z^i\}\,.$$

$T + \bar{T} = 2t$ appears as a non-propagating field in the Jordan frame.
If V_c is a quadratic function of t, then the theory is conformally equivalent to $\mathcal{R}^2$ gravity coupled to matter.

V_c is given in terms of the $Y(\psi^I)$ and the superpotential $W(\psi^I)$.

$$V_c = Y^2\, V_E$$

$$V_E = e^K \left\{ (W_I + K_I W)\, K^{I\bar{J}} \left(\bar{W}_{\bar{J}} + K_{\bar{J}} \bar{W}\right) - 3|W|^2 \right\} + D\mathrm{-terms}$$

K is the Kähler potential (a real function of the scalars), which defines the metric $K_{I\bar{J}}$ on the scalar manifold via its holomorphic and anti-holomorphic derivatives:

$$K_{I\bar{J}} = \frac{\partial}{\partial \psi^I} \frac{\partial}{\partial \bar{\psi}^{\bar{I}}} K(\psi^I, \bar{\psi}^{\bar{J}}) \,, \qquad \text{with} \quad K = -3 \log Y$$

$$K_{I\bar{J}} = -3 \frac{Y_{I\bar{J}}}{Y} + 3 \frac{Y_I \, Y_{\bar{J}}}{Y^2} \,, \qquad K^{I\bar{J}} \ \text{is the inverse of} \ K_{I\bar{J}}$$

The conformally equivalent action in the Einstein frame becomes :

$$S_E = \int d^4x \sqrt{|g|} \ \left[\frac{1}{2} \mathcal{R} - K_{I\bar{J}} \, \partial_\mu \psi^I \partial^\mu \bar{\psi}^{\bar{J}} - V_E \right]$$

OR

$$S_E = \int d^4x \sqrt{|g|} \ \left[\frac{1}{2} \mathcal{R} - \frac{3}{4} \frac{D_\mu D^\mu}{Y^2} - \frac{3}{4} \frac{J_\mu J^\mu}{Y^2} - 3 \left(\frac{-Y_{I\bar{J}}}{Y} \right) \partial_\mu \psi^I \partial^\mu \bar{\psi}^{\bar{J}} - V_E \right]$$

J_μ^2 is induced by the auxiliary vector field A_μ of the supergravity multiplet.

D_μ^2 is induced by the rescaling of the metric from the Jordan to the Einstein frame.

$$J_\mu = i(Y_{\bar I}\partial_\mu \bar\psi^{\bar I} - Y_I \partial_\mu \psi^I), \qquad D_\mu = Y_{\bar I}\partial_\mu \bar\psi^{\bar I} + Y_I \partial_\mu \psi^I \equiv \partial_\mu Y \, .$$

The D_μ kinetic part becomes the kinetic term of the no scale modulus ϕ

$$\alpha\phi = \log Y \quad \text{with} \qquad \alpha = \sqrt{\frac{2}{3}}$$

The J_μ kinetic part give rise to the kinetic term of $2b = i(\bar T - T)$, (modulo z^i-fibrations)

$$S_E = \int d^4x \sqrt{|g|} \left[\frac{1}{2}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - e^{-2\alpha\phi}\frac{3}{4}J_\mu J^\mu - e^{-\alpha\phi}\, 3\, \partial_\mu z^{\imath}\partial^\mu \bar z^{\bar\imath} - V_E\right]$$

- The scalar manifold is isomorphic to that of no-scale supergravity model $CP^{(1,n)}$

$$K = -3\log\left(T + \bar{T} - |z^i|^2\right) \longrightarrow \quad \mathcal{M}(T, z^i) = \frac{SU(1, 1+n)}{U(1) \times SU(1+n)}\,.$$

- The kinetic part is manifestly scale invariant, under :

$$T \to e^{2\sigma}T, \qquad z^i \to e^{\sigma} z^i \quad \longrightarrow \quad Y \to e^{2\sigma}Y, \quad e^{\alpha\phi} \to e^{2\sigma}e^{\alpha\phi}, \quad b \to e^{2\sigma}b.$$

- The scale symmetry is an invariance of the potential and also of the gauge and fermionic sectors, provided that the resulting supergravity **G** function

$$\mathbf{G} \equiv K + \log|W|^2\,, \qquad K = -3\log Y\,,$$

and the gauge kinetic holomorphic function $f_{ab}(\psi^I)$ are invariant.

- Under the scale transformation : $K \to K - 6\sigma$, imposing that :

$$W \to e^{3\sigma}W \quad \text{weight } 3$$

$$f_{ab} \to f_{ab} \qquad \text{weight } 0$$

- In the absence of fields with 0 scaling weight (or else) the choices for W and f_{ab} are strongly restricted. In particular f_{ab} is a constant :

$$f_{ab} = \frac{\delta_{ab}}{g_a^2}\,, \quad (\mathbf{i})\ W = c\,T^{3/2}, \quad (\mathbf{ii})\ W = c_{ijk}\,z^i z^j z^k, \quad (\mathbf{iii})\ W = c_k\,z^k\,T\,.$$

(i) $W = c\,T^{3/2}$

It gives rise to a scale invariant model with negative cosmological constant.

$\longrightarrow$ Anti de Sitter realization of the $-\mathcal{R}^2$ theory with unbroken supersymmetry.

The potential saturates the Freedman bound:

$$\mu^2 = V = -3m_{3/2}^2$$

(ii)$W = c_{ijk}\,z^i z^j z^k$

It is a particular case of a class of “no scale models”

$$Y = T + \bar{T} - g(z^i, \bar{z}^{\bar{j}})\,, \qquad W = W(z^i)\,.$$

with a “magic structure” similar to that of global supersymmetric theories:

$$V = \frac{3}{Y^2} W_i \, g^{i\bar{j}} \, \bar{W}_{\bar{j}} + V_D \, .$$

The potential is semi-positive definite independently if supersymmetry is preserved or broken. It depends only through the derivatives of W_i.

The well known negative contribution in supergravity is absent,

$$-3e^K |W|^2 = -3m_{3/2}^2$$

thanks to the $T + \bar{T}$ dependance of Y and the T-independence of W.

In the absence of non-trivial contribution from the D-terms, the T direction is flat, at any global minimum of the potential with $V = 0$, while the gravitino mass term is undetermined at the classical level.

When $W = c_{ijk} \, z^i z^j z^k$ it gives rise to scale invariant theory even with $W \neq 0$.

The classical potential is semi-positive definite and quartic in the fields z^i, modulo the dressing coming from the no-scale modulus ϕ $(1/Y^2 = e^{-2\alpha\phi})$:

$$V_F = e^{-2\alpha\phi} \left(z^4 \text{ terms} \right)$$

The F-part of the potential can never generate a non zero cosmological constant.
The only remaining way is via a non trivial Fayet-Iliopoulos D-terms.
In supergravity theory this can be achieved only and only if
W is transforming non-trivially under a local $U(1)$ R-symmetry.

$$z^i \to e^{iw}\, z^i, \qquad W \to e^{3iw}\, W$$

$$V = V_F\left(\hat{\Phi}_i\right) + V_D\left(|\hat{\Phi}_i|^2\right) \quad \longrightarrow \quad V = h^2\rho^4 + g^2(\rho^2 + 1)^2$$

$\rho^2 = |\hat{\Phi}_i|^2 = e^{-\alpha\phi}|z_i|^2$ is the "radial" scale invariant field. The 1 is the F-I term.
The "angle"-dependences in V_F are integrated over.
The effective $h^2 \neq 0$ in the non-degenerated case and $h^2 = 0$ in the degenerated case.

The classical theory is scale invariant. The vacuum solution is the de Sitter space time

$$\rho = 0\,, \quad \phi = \text{const.} \quad B = \text{const.} \quad (B = e^{-\alpha\phi}\, b)$$

(iii) $W = c_k\, z^k\, T \equiv c\, z\, T$

The specific T dependence, dressed by z, preserves the scale symmetry.
The T-dependence in W destroys the "no-scale structure" creating instabilities.
Here also a non-trivial contribution from V_D is necessary to stabilized the z direction.

$$V = V_F + V_D = h^2 \left(1 + 4B^2 - 2\rho^2 - 3\rho^4\right)_F + g^2 \left(3\rho^2 + 1\right)_D^2$$

$$V = (h^2 + g^2) + 4h^2\, B^2 + (3g^2 - h^2)\left(2\rho^2 + 3\rho^4\right)$$

At the extremum $B = \rho = 0$, V becomes constant with a cosmological constant $\mu^2 = (h^2 + g^2)$. The instabilities of V_F are resolved by V_D as soon as $(3g^2 - h^2) \geq 0$.

The classical vacuum is the de Sitter space-time with the modulus ϕ a flat direction.

The majority (totality) of the models proposed in the literature assumed a trivial D-terms. Many scenarios have been proposed in order to bypass the V_F instabilities:

(i) Changing the scale covariant Y function by introducing $|z|^4$ terms
(destroying randomly the scale-symmetry and the scalar manifold structure !)
(ii) Assuming that z is a composite fermion bi-linear with $z^2{=}0$!!
(iii) The total absence of W having only D-terms !!!

*** We will disregards all these proposals since almost all of them
do not seem to have any connection with the quantum superstring theory.

Even the classical "eternal de Sitter" constructions presented previously
are suffering by quantum anomalies (associated with the $U(1)_R$ gauging).

The anomalous $U(1)_R$ gauging has to be extended to a non-anomalous
$U(1)_t$ or $U(1)_d$ gaugings involving the isometries of the scalar manifold :

$$U(1)_t:\ T \to T + i\xi w, \quad z^i \to e^{iw} z^i \quad \text{OR} \quad U(1)_d:\ T \to e^{\xi w}\, T\,, \quad z^i \to e^{iw} z^i$$

The potential receives additional contribution which destabilized the classical dS vacuum inducing inflationary transitions towards to the flat Minkowski space-time :

(i) Exponentially suppressed in the $U(1)_t$, "Starobinsky like"

(ii) Polynomial transition in the $U(1)_d$, "two component chaotic inflationary models"

$$V_t = h^2\,\rho^4 + g^2\left(\rho^2 + 1 - \xi e^{-\alpha\phi}\right)^2_D$$

$$V_d = h^2\,\rho^4 + g^2\left(\rho^2 + 1 - \xi\, B\right)^2_D$$

In both non-anomalous gaugings, $U(1)_t$ and $U(1)_d$, the potential is semi-positive definite having global minima with $V_{d,t} = 0$.

The de Sitter vacuum becomes metastable due to the ξ-screening involving the no-scale modulus ϕ in V_t, OR in a scale invariant way involving B in V_d.

V_t has a structure similar to the Starobinsky potential.
*** In the Jordan frame the scale violating term, $-\xi e^{-\alpha\phi}$, is just the undressed $\xi\mathcal{R}$!

$$U(1)_t \ : \mathcal{R}^2 \longrightarrow \mathcal{R}^2 + \xi\mathcal{R}, \qquad \xi = \mathrm{Tr}\, Q_R$$

Q_R is the charge operator associated with the anomalous $U(1)_R$.

3. Superstring induced $\mathcal{R}^2$ models

In superstring theory the origin of the scale symmetry is purely geometrical. It follows from the initial 10-dimensional structure of the theory coupled to the dilaton ϕ_{10} and the antisymmetric tensor field $B_{\mu\nu}$.

In terms of the complex volume moduli T^A, the complex structure moduli U^A and the four dimensional complex dilaton field S,
the 4d effective action takes the following universal form:

$$S_E = \int d^4x \sqrt{|g|} \left[\frac{1}{2}\mathcal{R} - \frac{\partial_\mu S \, \partial^\mu \bar{S}}{(S+\bar{S})^2} - \frac{\partial_\mu T^A \, \partial^\mu \bar{T}^A}{(T^A+\bar{T}^A)^2} - \frac{\partial_\mu U^A \, \partial^\mu \bar{U}^A}{(U^A+\bar{U}^A)^2} + \ldots - V_F - V_D \right],$$

$$T^A = \sqrt{\det g^A_{ij}} + iB^A_{12}, \qquad U^A = \frac{g^A_{11} + i g^A_{12}}{\sqrt{\det g^A_{ij}}},$$

$$S = e^{-2\phi_4} + ia, \qquad e^{-2\phi_4} = e^{-2\phi_{10}} \sqrt{\det g_{IJ}}, \qquad \partial_\mu a = \epsilon_\mu^{\nu\rho\sigma} \, \partial_\nu B_{\rho\sigma}$$

This universal structure arises in all string compactifications, indicating some perturbative and non-perturbative equivalences between known string theories (String-String dualities) :

(a) Mirror symmetry : $T^A \leftrightarrow U^A$,

(b) Heterotic-type IIA non-perturbative string duality: $T^1 \leftrightarrow S$,

(c) Heterotic-type IIB non-perturbative string duality: $U^1 \leftrightarrow S$,

(d) Type IIA-type IIB orientifolds with D-branes and fluxes: $T^A \leftrightarrow U^A$, … .

Many of these moduli fields are frozen either by the compactification or by turing on various kind of fluxes.
In type IIB orientifold compactifications, S together with U^A can be frozen.
In type IIA orientifolds the situation is similar via the interchange: $U^A \leftrightarrow T^A$.

In the string effective supergravities the fluxes are in one to one correspondence with the gaugings of the graviphotons, appearing in sub-sectors of the $N = 1$ (or even $N = 0$) theory with non-aligned extended supersymmetries $N > 1$.

In the heterotic the S modulus is stabilized by non-perturbative effects, like for instance gaugino condensation and fluxes.
The remaining moduli in both heterotic and type II orientifolds are the geometrical volume moduli T^A. Their kinetic terms are invariant under the scalings

$$T^A \longrightarrow e^{2\sigma_A} T^A$$

and in particular, under the diagonal scaling where $\sigma_A = \sigma$.

This implies that the diagonal direction $T = T^A$ appears always with a universal normalization coefficient: (the one of the no-scale models)

$$-\frac{\partial_\mu T^A \, \partial^\mu \bar{T}^A}{(T^A + \bar{T}^A)^2} \longrightarrow -3 \, \frac{\partial_\mu T \, \partial^\mu \bar{T}}{(T + \bar{T})^2}, \quad \text{with} \quad K = -3 \log(T + \bar{T})$$

In the Jordan frame, $T + \bar{T}$ appears without kinetic term :

$$\frac{1}{2}(T + \bar{T}) \left(\mathcal{R} + \frac{2}{3} A_\mu A^\mu \right) - J_\mu A^\mu + \ldots - (T + \bar{T})^2 (V_F + V_D)_E \, .$$

When the potential in the Einstein frame is $(T + \bar{T})$ independent, then the algebraic equation of $(T + \bar{T}) \longrightarrow$ the theory is conformally equivalent to $\mathcal{R}^2$ theory.

Switching on all other degrees of freedom associated with the internal metric, and the other chiral fields, the above property remains valid.

In general the diagonal $T + \bar{T}$ mode is related to the volume of the six dimensional compact manifold and its fluctuations:

$$(T + \bar{T}) \quad \longrightarrow \quad Y = e^{-\frac{1}{3}K(T^a\,,\,z^I\,)}\,.$$

T^a stand for all components of the metric (the $h_{1,1}$ moduli fields)
z^I denote the Wilson line moduli, as well as the chiral matter fields arising from :
– The orbifold twisted states in the heterotic string, and
– Localized states on D_3 and D_7 branes in the Type II orientifolds
It is remarkable that Y transforms homogeneously under the scale transformations:

$$T^a \rightarrow e^{2\sigma}\,T^a, \quad z^I \rightarrow e^{\sigma}\,z^I, \quad Y \rightarrow e^{2\sigma}\,Y \quad \text{and} \quad K \rightarrow K - 6\,\sigma\,.$$

States with scaling weight $w = \mathbf{3}$, can also appear. Their presence however is always compatible with the above transformation properties of Y and K.

The profound reason of the scale symmetry has its origin in the modular invariance of string theory. Remarkably, the above property of the geometrical T-sector extends to the U-sector, and to several STU-sectors thanks to Sting-String dualities.

Equally significant is the presence of several anomalous $U(1)_R$-symmetries.
These anomalous symmetries are always corrected at the string level
via local axion gaugings which promotes: $U(1)_R \longrightarrow U(1)_t$ or $U(1)_d$

4. Typical superstring inflationary models

In the heterotic on orbifolds and type IIB on orientifolds with stabilized S and U^A
the typical Kähler potential depends on the three main moduli T_A:

$$K = -\log Y_1 - \log Y_2 - \log Y_3, \qquad Y_A = (T_A + \bar{T}_A) - |z^i_A|^2 .$$

4.1 Typical superstrings with trilinear superpotentials

$$W = d^{ABC}_{ijk}\, z^i_A\, z^j_B\, z^k_C\, . \quad A, B, D = 1, 2, 3$$

There are three distinct $U(1)^A_R$ R-symmetries, one for each complex plane.
From the effective supergravity point of view, it is a matter of choice the gauging
either all of them, one or two of them, or even diagonal combinations of those.

**** In string theory however this is not a choice !
It follows from the consistency of the string spectrum and interactions.
Once there is a net chirality emerging in a given sector A there is an anomalous $U(1)_R^A$.
The promotion of $U(1)_R^A$ to the non-anomalous $U(1)_t^A$ is a stringy necessity.

$$V = V_F + \sum_{A \text{ chiral}} g_A^2 (|\rho_A|^2 + 1 - \xi_A \, e^{-\gamma \phi_A})^2 \,, \qquad \gamma = \sqrt{2}$$

V_F is the universal F-term contribution to the potential. The slow roll inflationary transition is transparent in terms of the canonically normalized fields ϕ_A:

$$Y_A = e^{\gamma_A \phi_A}, \quad \gamma_A = \sqrt{2} \,.$$

The over all no-scale modulus ϕ is the diagonal modulus

$$Y = (Y_1 Y_2 Y_3)^{\frac{1}{3}} = e^{\frac{\gamma}{3}(\phi_1 + \phi_2 + \phi_3)} = e^{\alpha \phi} \,, \qquad \alpha = \sqrt{\frac{2}{3}} \,,$$

as expected by the conformal equivalence with the $\mathcal{R}^2$ theory in the Jordan frame.

If V_F has no flat directions, there is a unique global flat vacuum with: $\Phi^I_A = 0$ and all ϕ_A moduli stabilized. The structure of the off-shell potential is similar to that of the simple Starobinsky model generalized for two and three inflaton fields.

If V_F is flat in the Φ directions, the flat vacuum is degenerate with: $D^A_t = 0$. ϕ_A and Φ^I_A acquiring non trivial vacuum expectation values.
The vacuum degeneracy will be lifted at quantum level
especially when supersymmetry is spontaneously broken.

In the de Sitter and inflationary eras, the ξ_A scale violating effects are exponentially suppressed. The fluctuations of Φ^I_A are massive. Their masses are protected by the induced de Sitter mass even at the quantum level.

4.2 Typical superstrings with linear superpotentials

Superstring with geometrical fluxes permit linear superpotentials in both the heterotic and type IIB cases, even though their constructions are non trivial. The generic F part of the potential has pathological behavior. This pathology is due to the appearance of T^A moduli in W destroying the “no-scale” positivity properties.
This pathological behavior is cured in certain cases by FI-D-terms.

The simplest cases is when one of the z-fields is turned on in the first complex plane :

$$(\mathbf{a})\ W = h\, z\, (T_2 - T_3)\,, \qquad (\mathbf{b})\ \ W = h\, z\, (T_2 + T_3)$$

Effective linear superpotentials with spontaneously broken the scale invariance, can also arised in superstings with non-trivial geometrical fluxes.
The breaking is induced by the “freezing mechanism” which stabilizes the $U^A \neq 0$.

$$(\mathbf{c})\ W = h\, z(2T - \xi), \quad \text{with} \qquad T = T_2 = T_3$$

(a) $W = h\, z\, (T_2 - T_3)$

$$V = h^2 \left(B_2\, e^{\frac{\gamma}{2}(\phi_2 - \phi_3)} - B_3\, e^{-\frac{\gamma}{2}(\phi_2-\phi_3)} \right)^2 + h^2\, \mathrm{sh}^2 \left(\frac{\gamma}{2}(\phi_2 - \phi_3) \right)$$

$$+ 2h^2 |\Phi|^2 + g^2 \left(\, |\Phi|^2 + 1 - \xi_1 e^{-\gamma\phi_1} - \xi e^{-\gamma\phi_2} - \xi e^{-\gamma\phi_3} \, \right)^2_D \ ,$$

(b) $W = h\, z\, (T_2 + T_3)$

$$V = h^2 \left(B_2\, e^{\frac{\gamma}{2}(\phi_2 - \phi_3)} + B_3\, e^{-\frac{\gamma}{2}(\phi_2-\phi_3)} \right)^2 + h^2\, \mathrm{ch}^2 \left(\frac{\gamma}{2}(\phi_2 - \phi_3) \right)$$

$$- 2h^2 |\Phi|^2 + g^2 \left(\, |\Phi|^2 + 1 - \xi_1 e^{-\gamma\phi_1} - \xi e^{-\gamma\phi_2} + \xi e^{-\gamma\phi_3} \, \right)^2_D$$

In the case **(a)** the F part of V is well behaving, without instabilities in the Φ direction.
The potential is scale invariant modulo the ξ-anomaly terms.
V is semi-positive definite and has a global flat vacuum:

$$\Phi = 0, \quad \xi_1\, e^{-\gamma\phi_1} + \xi\, e^{-\gamma\phi_2} + \xi\, e^{-\gamma\phi_3} = 1$$

From a cosmological view point it can be analyzed as a two component inflation, since the potential attracts $\phi_3 \to \phi_2$ and $B_3 \to B_2$ rapidly.

In the case (b) the F part of the potential is unstable in the Φ direction.
V_D resolves the tachyonic behavior during the inflationary era as soon as $g^2 - h^2 \gg 0$
The global vacuum has runaway behavior because of the scale breaking terms.
Most models of this type have similar behavior and has to be disregarded.

(c) $W = h\, z_1\, (2T - \xi), \quad$ with $\quad T = T_2 = T_3$

$$V = 4h^2 B^2 + h^2 \left(1 - \xi e^{-\phi_2}\right)^2 + h^2 \left(4\xi e^{-\phi_2} - 2\right) |\Phi|^2 + g^2 \left(|\Phi|^2 + 1 - \xi_1 e^{-\gamma\phi_1}\right)^2_D$$

The F part of the potential is by itself pathological, as in the Cecotti like models proposed in the litterature. This instability is resolved by V_D in the de Sitter era as soon as $g^2 > |h|^2$. There is a stable minimum with V=0 :

$$1 - \xi e^{-\phi_2} = 0, \quad 1 - \xi_1 e^{-\gamma\phi_1} = 0, \quad B = 0, \quad \Phi = 0 \ \text{ with } \ V = 0$$

In the region where the scale breaking terms have asymmetric behavior with:

$\xi_1 e^{-\gamma\phi_1}$ dominant and $\xi e^{-\phi}$ subdominant, there exist unstable directions in V.

In these directions the D-term can vanish with $|\Phi|^2$ non-trivial.
Along these directions the potential is unbounded from below when:

$$4\xi e^{-\phi_2} < 2 \qquad \text{and} \qquad \xi_1 e^{-\gamma\phi_1} \gg 1$$

The asymmetric destabilization can be avoided via suitable geometrical fluxes enforcing the diagonal T-directions, $T \sim T_1$.

5. Conclusions

- $\mathcal{R}^2 \oplus$ Conformal matter including all possible interactions.
  We obtain a classical scale-invariant theory with an extra degree of freedom ϕ (the no-scale modulus) defined on $H^{n+1} \equiv SO(1,1+n)/SO(1+n)$ scalar manifold.
- Classically there are three and only three possible vacua according to the coupling μ^2 of the $\mathcal{R}^2$ theory: dS ($\mu^2 > 0$), $AdS(\mu^2 < 0)$ and flat $\mathcal{M}$ ($\mu^2 \to 0$).
- The undressed $\xi\mathcal{R}$ scale-violating term is the origin of an inflationary slow roll transition from the de Sitter to the flat space.
- In the minimal supersymmetric extension the scalar manifold becomes Kählerian $\mathcal{M} = CP^{(1,n)} \equiv SU(1,1+n)/U(1) \times SU(1+n)$
  This is the one of the minimal no-scale supergravities with the "magic" positivity properties of $V_F \geq 0$ once the no-scale modulus T does not appear in W.
- The only natural way to resolve the instabilities of a T-dependent W is by non-trivial FI D-terms obtained by gauging $U(1)_R$ symmetries (anomalous !).

- $U(1)_R$ has to be extended in a consistent way by gauging simultaneously some of the isometries of the scalar manifold involving the dilatation or the axionic symmetry. $U(1)_R \longrightarrow \;\; U(1)_d$ and $U(1)_t$ non-anomalous gaugings.
- In both $U(1)_d$ and $U(1)_t$ gaugings the classical de Sitter vacuum is destabilized either in a rapid or slow way giving in both cases inflationary behavior during the transition to the flat Minkowski space-time.
- Metastable de Sitter vacua are present in all chiral $N = 1$ superstring models. The superstring resolution of these anomalies are achieved via local axion shifts promoting the several $U(1)^A_R \rightarrow U(1)^A_t$ or $U(1)^A_d$ non-anomalous local symmetries.

Thank you for your attention

Costas Kounnas

Works in collaboration with E. G. Floratos (∼ 600 citations)

- E. G. Floratos, C. Kounnas and R. Lacaze, *"Higher order QCD effects in Inclusive annihilation and deep inelastic scattering,"* Nucl. Phys. B **192** (1981) 417.
- E. G. Floratos, C. Kounnas and R. Lacaze, *"Space and timelike cut-vertices in QCD beyond the leading order 1. Nonsinglet sector,"* Phys. Lett. B **98** (1981) 89.
- E. G. Floratos, C. Kounnas and R. Lacaze, *"Space and timelike cut-vertices in QCD beyond the leading order 2. The singlet sector,"* Phys. Lett. B **98** (1981) 285.
- L. Baulieu, E. G. Floratos and C. Kounnas, *"Higher order corrections in the cut-vertex formalism,"* Phys. Rev. D **23** (1981) 2464.
- L. Baulieu, E. G. Floratos and C. Kounnas, *"Parton model interpretation of the cut-vertex formalism,"* Nucl. Phys. B **166** (1980) 321.
- L. Baulieu, E. G. Floratos and C. Kounnas, *"Crossing relations for deep inelastic and annihilation processes,"* Phys. Lett. B **89** (1979) 84.

Personal experiences with Manolis

I met Manolis in1969 when I started my undergraduate studies at the University of Athens. Manolis was then a graduate student in theoretical physics. We became very good friends. I learn a lot from him by interacting scientifically. When I was in the 4th year of my studies Manolis became one of my best teachers (together with Thanasis Lahanas)!

We went in the army at the same time, 1973-1976, (the worse period one can imagine), during the Turkish invasion in Cyprus. Both of us served in the army for more than 30 months.

After the army, we met again in Paris and we started collaborating. At that epoch, 1976-1980, I was a graduate student while Manolis was Fellow at CERN, after a post-doc in Saclay and then in ENS. During that period we were not only collaborators, but also very close friends.

Manolis achievements

Manolis is one of the creators of the University of Crete.
He upgraded the Institute of Particle Physics at Democritos when he was Director.
Furthermore, he upgraded the CERN-GREECE relations, in theory, in experiment and in Information Technology, as the Greek representative at CERN.

His contribution in advancing particle and theoretical physics in Greece is really tremendous ! I was at CERN as a staff at that epoch (1992-1999) and I assure you that his efforts were enormous, given the endless financial and other administrative problems in Greece.

We found the excuse to be here for his retirement. For me this is absolutely wrong ! Manolis will never retire ! We are here to thank him for his contribution to physics and for his efforts in advancing physics in Greece but also in Europe.

Thank you my friend