QCD Spectral Sum Rules for Hadrons

Floratosfest 2014

Athens 10th October 2014

Stephan Narison

CNRS / IN2P3 - LPUM (Montpellier)

Contents

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- ♣ Floratos Visit at CPT-Marseille (1978)
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- ♣ Theoretical progresses
- ♢ Traditional and New Hadron phenomenology
- ♡ SVZ Power Corrections

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♣ Decay Constants

♢ Heavy Exotic Hadrons

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- ♣ Decay Constants
- ♢ Heavy Exotic Hadrons
- ♡ Conclusions

Floratos Visit at CPT-Marseille (1978)

- Former CPT-Marseille site Joseph Aiguier

Floratos Seminars - Marseille (1978)

SEMINARS ON SCALING VIOLATIONS

IN QCD

MARSEILLE - CNRS / CPT

MARS 1978.

PROGRAMME OF THE SEMINARS

1st Lecture

1. Introduction and the reason for the calculation in order a_s
2. Operator product expansion and the moment relations for structure functio
3. Dimensional renormalization scheme, renormalization group eq and the anomalous dimension of Wilson operators

2nd Lecture

4. Resolution of the pseudo problem of renormalization scheme dependence of the moments
5. The calculation of the anomalous dimension of the non singlet operator and the problem of overlapping divergences

3rd Lecture

6. The problem of mass corrections.
7. Modified moment sum rules

4th Lecture

8. The moment inversion. Comparison of different contributions.
9. Comparison with the experiment and open problems.

Collaboration with Floratos in Marseille

Nuclear Physics B155 (1979) 115–149

SPECTRAL FUNCTION SUM RULES IN QUANTUM CHROMODYNAMICS (I). Charged currents sector

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Received 7 August 1978
(Revised 2 March 1979)

The Weinberg sum rules of the algebra of currents are reconsidered in the light of quantum chromodynamics (QCD). We derive new finite-energy sum rules which replace the old Weinberg sum rules. The new sum rules are convergent and the rate of convergence is explicitly calculated in perturbative QCD at the first non-trivial order. Phenomenological applications of these sum rules in the charged current sector are also discussed.

♣ Breaking of the Weinberg Sum rules by Quark masses

- Superconvergent Weinberg Sum rules for $m_q = 0$

Asymptotic realization of $SU(2)_L \otimes SU(2)_R$ chiral symmetry.

$$\int_0^\infty dt \left[\mathrm{Im}\,\Delta^{1+0}(t) \equiv F_V^{1+0}(t) - \mathrm{Im}\,F_A^{1+0}(t)\right] = 0$$

$$\int_0^\infty dt\; t \times \left[\mathrm{Im}\,\Delta^{1,0}(t) \equiv \mathrm{Im}\,F_V^{1,0}(t) - \mathrm{Im}\,F_A^{1,0}(t)\right] = 0$$

Difference between the vector and axial-vector two-point correlators vanishes at large $-q^2 \equiv Q^2 > 0$ which obeys the dispersion relation:

$$F_{V,A}(q^2) = \int_0^\infty \frac{dt}{t - q^2 - i\varepsilon}\, \frac{1}{\pi}\mathrm{Im}\,F_{V,A}(t) + \ldots \quad : \quad F\ like\ Floratos\ ?$$

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- Mass Corrections

$$\mathrm{Im}\,\Delta_{\mathrm{ij}}^{1,0}(\mathrm{t}) = \pm\frac{3}{2\pi}\frac{m_i m_j}{t}\left\{1 - \frac{m_i^2 + m_j^2}{t} - 2\frac{m_i^2 m_j^2}{t^2}\ldots\right\}$$

$$\Delta^{1+0}(Q^2 \gg m_q^2) = \left(\frac{\alpha_s}{\pi}\right)\frac{m_i m_j}{q^2}\frac{1}{\pi^2} + \ldots \qquad (2)$$

♢ Improved Weinberg Sum Rules for $m_q \neq 0$

$$\mathrm{Im}\, R^{1,0}_{ijk} \equiv \Delta^{1,0}_{ij} - \frac{m_j}{m_k}\Delta^{1,0}_{jk}$$

$$\mathrm{Im}\, R^{1,0}_{ijkl} \equiv \mathrm{Im}\, R^{1,0}_{ijk} - \frac{m_i}{m_l}\mathrm{Im}\, R^{1,0}_{ljk}$$

See also:

C. Bernard, A. Duncan, J. Losecco, S. Weinberg (1975)

T. Hagiwara, R. Mohapatra (1975).

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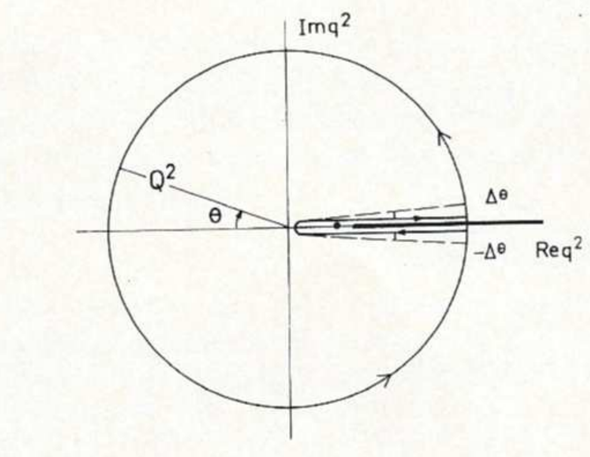
♡ Needs for a RGI Quark Mass in the $\overline{MS}$-scheme:

$$\hat{m} = m(\mu)\left(\frac{\pi}{-\beta_1\alpha_s(\mu)}\right)^{\gamma/-\beta_1} \quad : \quad \overline{m}(Q) = \frac{\hat{m}}{[\log(Q/\Lambda)]^{\gamma_1/-\beta_1}}$$

$\gamma_1 = 2\,, \quad -\beta_1 = -(1/2)(11 - 2n/3)$

$\hat{m}$: scale invariant analogue to Λ.

◇ Finite Energy Weinberg Sum Rules for $m_q \neq 0$



Choice of integration contour in the complex q^2 plane

$$\int_0^{Q^2} dt\, t^p\, \mathrm{Im}\,\Delta(t) = \frac{(-1)^p (Q^2)^{p+1}}{2\pi} \int_{-\pi}^{+\pi} d\theta\, e^{i(p+1)\theta} \Delta(-Q^2 e^{i\theta}) \tag{3}$$

Various phenomenological applications

$M_{A_1} \geq$ 1045(16) MeV

$m_s/m_d \geq (M_K f_K)^2/(m_\pi f_\pi)^2 = 16.8$

...

QCD spectral sum rules (QSSR)

- One of the Outstanding Discoveries of the 20th century

  Shifman Vainshtein Zakharov 1979 (Sakurai's price 1999)

  QSSR instead of the popular QCD sum rules:

  - Spectral important as we work with Spectral functions
  - QSSR near the former Soviet Union state USSR where comes SVZ

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♢ The example of Exponential / Borel / Laplace sum rule

$$\mathcal{L}(\tau) = \int_{t_<}^{\infty} dt\, \exp^{-t\tau}\, \mathrm{Im}\Pi(t) \qquad \mathcal{R}(\tau) \equiv -\frac{d}{d\tau}\log\mathcal{L}(\tau)$$

- Improvement of the previous dispersion relation

$$\Pi_H(Q^2 \equiv -q^2) \equiv i\int d^4x\, \langle 0|\mathcal{T} J_H(x) J_H^\dagger(0)|0\rangle = \int_{t_<}^{\infty} \frac{dt}{t+Q^2+i\varepsilon}\mathrm{Im}\Pi(t) + \ldots$$

$$J_H(x) \quad : \quad \text{Hadronic current : } \bar\psi\Gamma\psi,\ \psi\psi\psi,\ \alpha_s G^2,\ g\bar\psi G\psi,\ \bar\psi\Gamma_1\psi\bar\psi\Gamma_2\psi, \ldots$$

- Bridge between QCD High AND Experimental Low energy regions: QCD OPE=EXP DATA
- Exponential enhances the low energy contribution.

The SVZ-OPE: Adler function in e^+e^-

- $D(Q^2)$ known in QCD using the SVZ-OPE

$$D(Q^2) \equiv -Q^2 \frac{d}{dQ^2} \Pi(Q^2) = \sum_{p=0,1,2,\ldots} \frac{C_{2p} \langle 0 | O_{2p} | 0 \rangle}{Q^{2p}}$$

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♢ Anatomy of the OPE in terms of QCD parameters

- $p=0$: usual PT series $(a_s \equiv \alpha_s/\pi)$:
  $C_0 \equiv \sum_n c_n \alpha_s^n = 1 + a_s + 1.640\ a_s^2 + 6.371\ a_s^3 + 49.076\ a_s^4 +$
  $\Delta_N \equiv \sum_{n>N} c_n \alpha_s^n$??
- $p=1$: $\overline{m}_q^2$: small corrections
- $p=2$: $\langle \alpha_s\, G^2\rangle$, $m_q\langle \overline{\psi}_q \psi_q\rangle$: gluon and quark condensates
- $p=3$: $m_q\langle g\overline{\psi}G\psi\rangle$: mixed quark-gluon condensate
  $\alpha_s\langle \overline{\psi}_q \psi_q\rangle^2$: four-quark condensates

Theoretical Progresses

- 1st Step
  - Radiative corrections into Exponential SR :

    Inverse Laplace NOT Borel transform SN- de Rafael 1981

Theoretical Progresses

1st Step

- Radiative corrections into Exponential SR :

  Inverse Laplace NOT Borel transform SN- de Rafael 1981

2nd Step

- Mixing of operators under renormalization
  Tarrach 82, Espriu-Tarrach 82, SN-Tarrach 83,...
- Absorption of Mass singularities into the quark condensate
  Becchi-SN-de Rafael-Yndurain 81, Broadhurst-Generalis 84, Bagan - Latorre - Pascual 86, BNP 92, Jamin-Munz 95, Chetyrkin-Steinhauser 01
- High-dimension gluon condensates
  Nikolaev-Radyushkin (NR) 83, BG 84, Bagan-Pascual-Tarrach 85,...
- Higher order PT corrections Chetyrkin et al...
- Duality between large order terms: $\Delta_N \equiv \sum_{n>N} c_n \alpha_s^n$ and a short distance $1/Q^2$ term due to a tachyonic gluon mass
  Chetyrkin-SN-Zakharov 95, SN-Zakharov 09

Spectral Function

♣ 1st Step: Available Data

- $e^+e^- \to$ hadrons and τ-decays (Axial)-vector channels

Spectral Function

♣ 1st Step: Available Data

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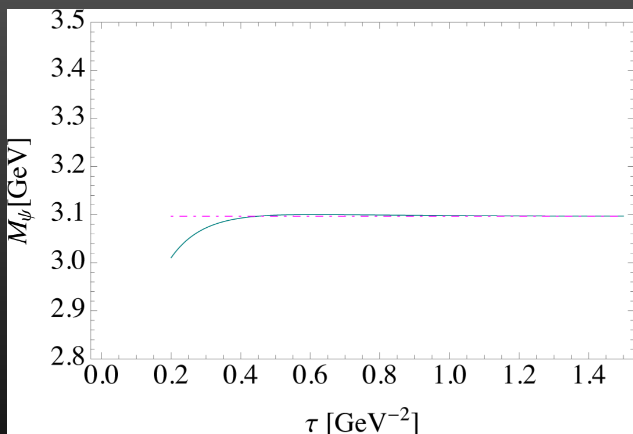
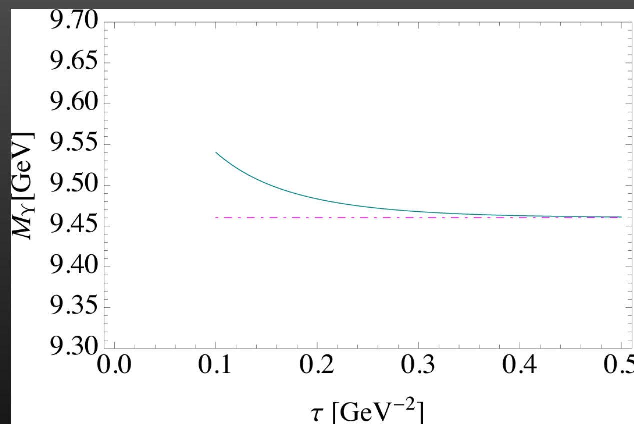
♡ 2nd step: No Data

- Minimal Duality Anstaz
  One resonance + QCD continuum
- Tested from $e^+e^{\to}$ I=1 hadrons data SN-WSC book 89
- Tested from charmonium and bottomium data SN12



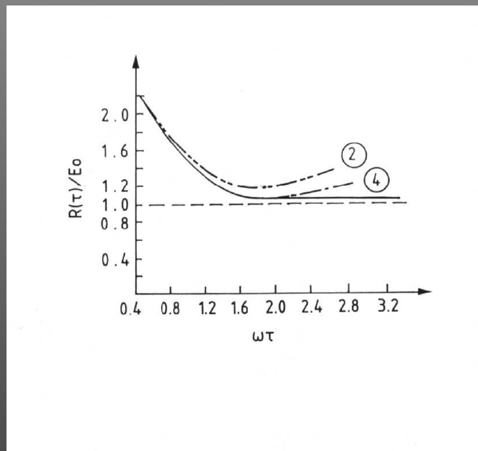


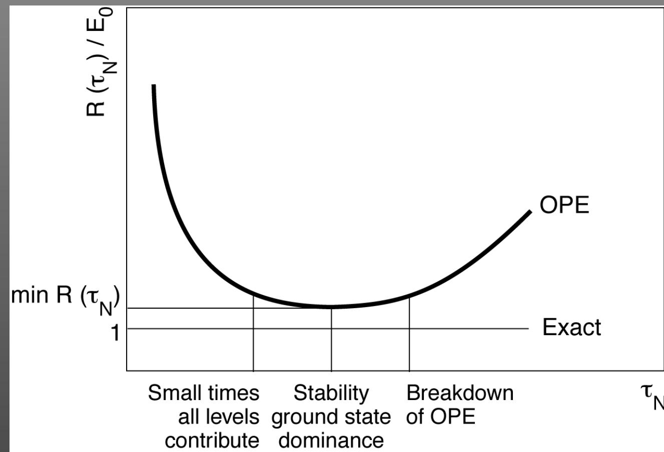
- Tested from large N_c-expansion Peris, Phili, de Rafael

Optimization Procedure

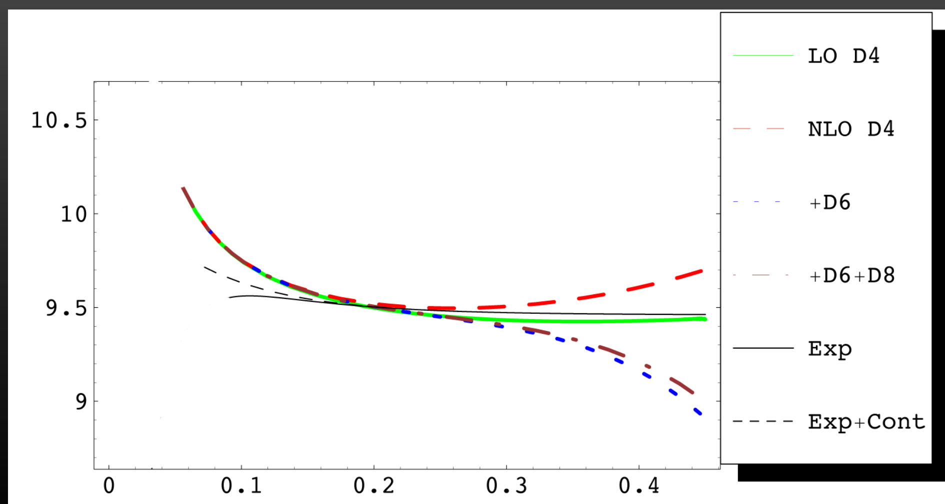
♣ 1st Step: τ-stability / Sum Rule Window

- Harmonic Oscillator & Non-Rel. SR Bell-Bertlmann (BB), NSVZ 81



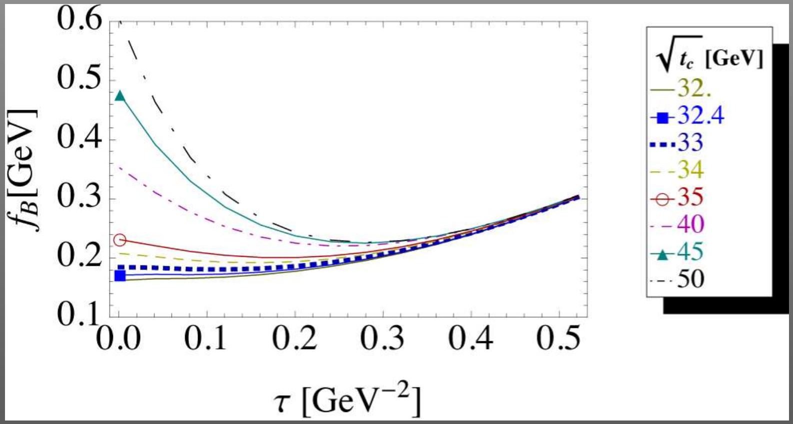


- Bottomium SN10



♣ 2nd Step: t_c-stability: conservative result

- Example of the decay constant f_B SN 12



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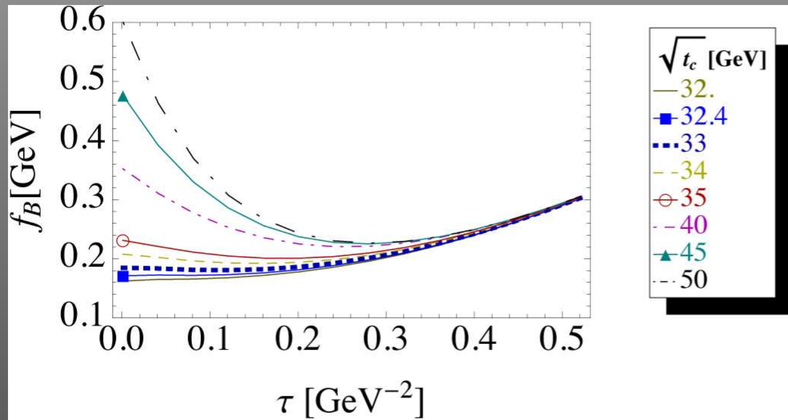
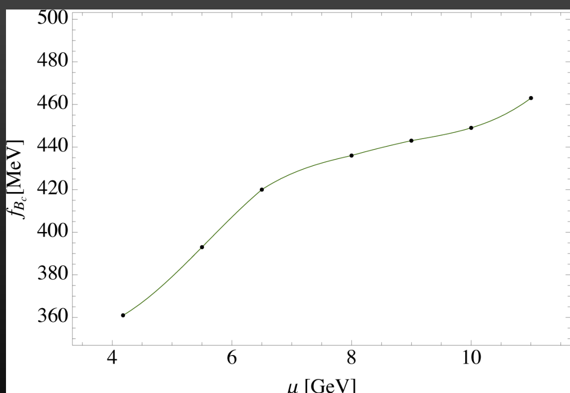
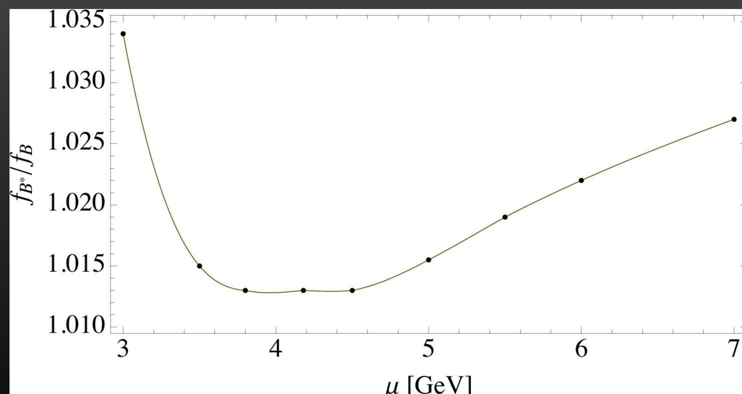


♡ 3rd step: μ-subtraction point stability

- Example of f_{B_c} and f_{B^*}/f_B SN 14





Traditional Hadron Phenomenology

♣ Impressive wide areas of applications: kalashnikov !

- ρ meson, gluon condensate, charm mass since SVZ 1979
- Meson spectroscopy since Reinders-Rubinstein-Yazaki 81
- Light quark masses since Becchi-SN-de Rafael-Yndurain 81, SN-Paver-de Rafael-Treleani 83,...
- Corrections to π and K PCAC since SN 81
- Heavy quark masses since SVZ 79, SN 87, Chetyrkin et al, Ioffe-Zyablyuk, SN 10,...,
- Light Baryons since Ioffe 81, Dosch et al 81
- Heavy Baryons since Bagan-Chabab-Dosch-SN 92, Albuquerque, Nielsen
- Gluonium since Novikov SVZ 80, SN 83, SN-Veneziano 89
- Light Hybrids since Latorre-Pascual-SN 87, Chetyrkin-SN 00, SN 09
- Heavy Hybrids since Govaerts-R-R-Weyers 85, Steele et al 2012
- Four-quarks, molecules : Light : since Latorre-Pascual 85, SN 86
  Heavy : Sao Paulo, Madagascar, China
- Hadronic decays Vertex Navarra et al.; Light Cone
- τ-decay since BNP 92, Pich-Lediberder, Prades-Pich,...
- Thermal Hadron Bochkarev-Shaposhnikov 86, Dosch-SN 88, Loewe, Buchheim

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♡ SN books : WSC 89, Cambridge 04

SVZ Power corrections SNB 04 Chap. 27

♣

d	NPQCD param.	Values [GeV]^d	Sources
2	$(\alpha_s/\pi)\lambda^2$	$-(7\pm3)\times10^{-2}$ GeV^2	SNB 04 Chapter 30
3	$\frac{1}{2}\langle\bar{u}u+\bar{d}d\rangle(2)$	$-(0.254\pm.015)^3$	(pseudo)scal,
	GMOR: $\delta_\pi\times10^{-2}$	5 ± 0.5 (6.2 ± 1.6)	LSR (less sens α_s) (FESR)
	$\langle\bar{d}d\rangle/\langle\bar{u}u\rangle$	$1-9\times10^{-3}$	non-norm. ord.(pseudo)scal,
	$\langle\bar{s}s\rangle/\langle\bar{d}d\rangle$	0.74 ± 0.03	non-norm. ord.(pseudo)scal
			$\oplus$ light & heavy baryons
4	$\langle\alpha_s G^2\rangle$	$(7\pm1)10^{-2}$	$e^+e^-,\ \Upsilon-\eta_b,\ J/\psi$ Laplace
	Lattice	Adriano 85, Rakow 09	τ, J/ψ mom : unconclusive
5	$g\langle\bar{\psi}\sigma_{\mu\nu}\frac{\lambda^a}{2}\psi G_a^{\mu\nu}\rangle$	$M_0^2=(0.80\pm0.02)$	Light baryons, $B,\ B^*$
	$\equiv M_0^2\alpha_s^{1/3\beta_1}\langle\bar{\psi}\psi\rangle$		
6	$g^3 f_{abc}\langle G^aG^bG^c\rangle$	$(8.2\pm2.0)\langle\alpha_s G^2\rangle$	J/ψ-mom SN10
	$\rho\alpha_s\langle\bar{\psi}\psi\rangle^2$	$(4.5\pm0.3)10^{-4}$	$\rho=2.1\pm0.2$

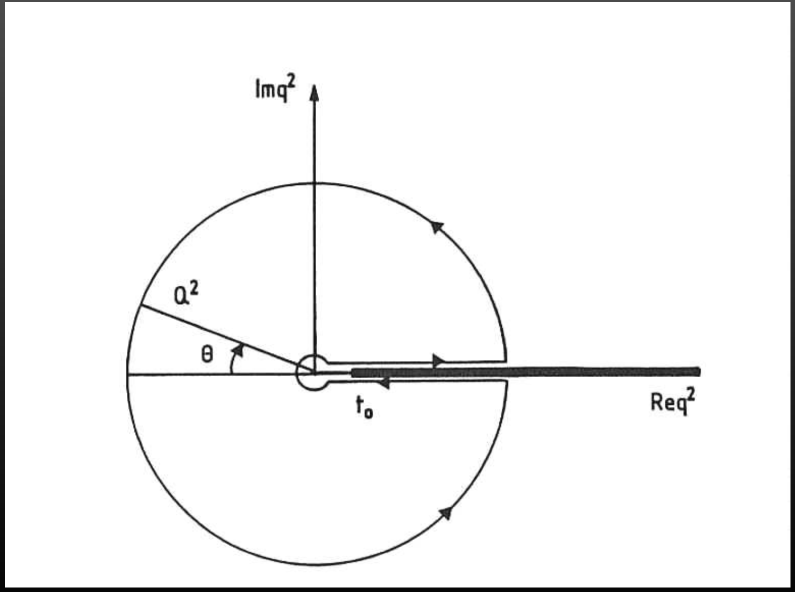
α_s from τ-decay at N4LO $\oplus$ $1/Q^2$

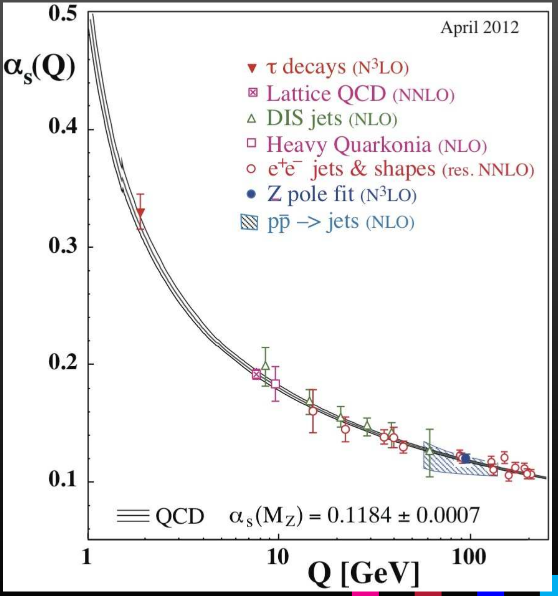
◇ Based on BNP 92 (Cauchy contour)

Param	Values	Sources
$\alpha_s(M_Z)$	$0.1192(10)$	τ-decay N4LO+Power Corrections SN 09
		0.1191(27) Z-width
		0.1184(7) average (Bethke, PDG)

Cauchy contour

α_s determinations as function of Q

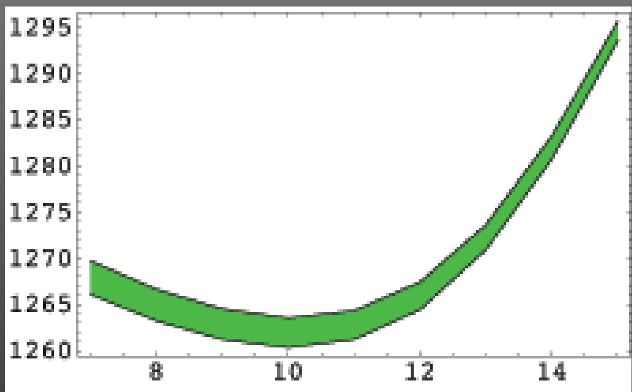


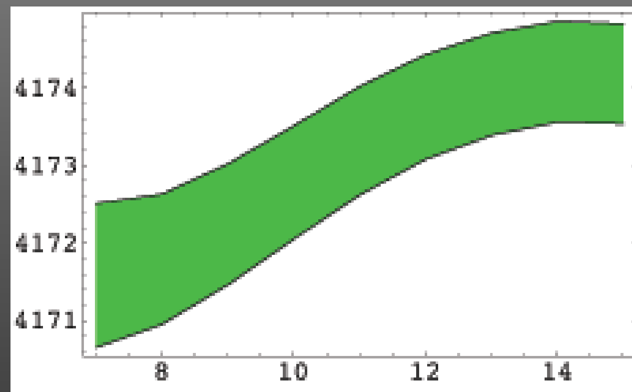


Charm & Beauty Quark Masses at N3LO

♣ $\text{Mom}: \mathcal{M}_n(Q^2) = \int_{4m_Q^2}^{t_c} dt \, \frac{1}{(t+Q^2)^{n+1}} \, \text{Im}\Pi(t)$ $\text{LSR}: \mathcal{L}(\tau) = \int_{4m_Q^2}^{\infty} dt \, \exp^{-t\tau} \, \text{Im}\Pi(t)$

$\overline{m}_c(m_c)$ as function of the degree of $\mathcal{M}_n$ for $Q^2 = 4m_c^2$ The same for $\overline{m}_b(m_b)$

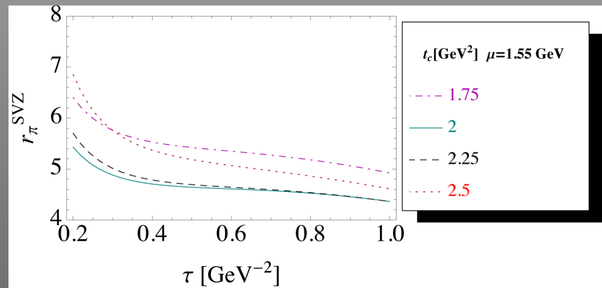


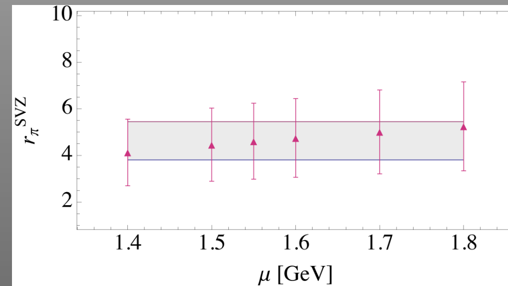


Param	Mean Values	Sources	
$\overline{m}_c(m_c)$	(1261 ± 12)	SN10,12	J/ψ, D Moment & Laplace SR
$\overline{m}_b(m_b)$	(4177 ± 11)	SN10,12	Υ, B Moment & Laplace SR

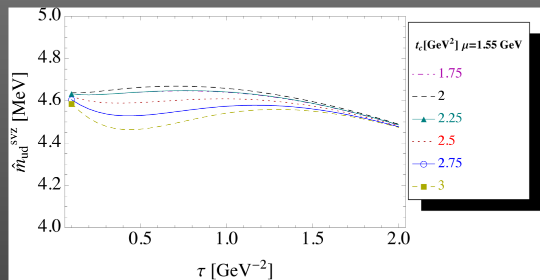
$\overline{m}_{u,d,s}$ from Pseudoscalar LSR at N4LO $\oplus$ $1/Q^2$

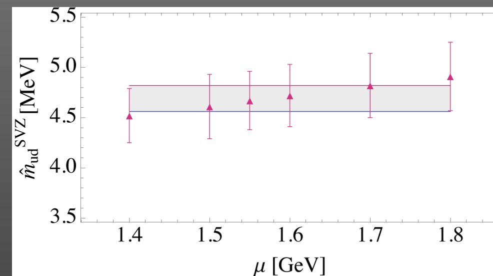
♣ $r_\pi = \left(\frac{M^2_{\pi'} f_{\pi'}}{m^2_\pi f_\pi}\right)^2$





♦ $\overline{m}_{ud} \equiv \frac{1}{2}(\overline{m}_u + \overline{m}_d)$





Param	Values [MeV]	Sources
$\overline{m}_u(2\ \text{GeV})$	(2.64 ± 0.28)	SN14 Pseudoscalar Laplace SR
$\overline{m}_d(2\ \text{GeV})$	(5.27 ± 0.49)	"
$\overline{m}_s(2\ \text{GeV})$	(99.0 ± 5.5)	" $\oplus$ e^+e^- $\oplus$ τ-decay

QSSR Results

Channels	Results [MeV]	Bounds	Sources
f_D	204(6)	$\leq 218(2)$	N2LO SN12
f_B	206(7)	$\leq 235(4)$"	
f_{D_s}	246(6)	$\leq 246(6)$	"
f_{B_s}	234(5)	$\leq 251(6)$	"
f_{D^*}	246(7)	$\leq 274(3)$	N2LO SN14
f_{B^*}	212(8)	$\leq 270(3)$	"
f_{B_c}	401(14)	$\leq 469(5)$	"

Competitive compared with Lattice calculations !

Charm and Beauty Exotics

- Existing LO results $\oplus$ 1 class of Power Corrections

Sao Paulo (Albuquerque, Navarra, Nielsen talks) & Chinese groups

Charm and Beauty Exotics

- ♣ Existing LO results $\oplus$ 1 class of Power Corrections

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- ♢ Needs to be improved: quark mass at LO ill-defined : running $\equiv$ pole mass !

Charm and Beauty Exotics

- ♣ Existing LO results $\oplus$ 1 class of Power Corrections

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- ♡ First calculation of the Heavy-light molecule masses at N2LO

  Fenosoa & Rabemananjara $\oplus$ SN 14:

  - Small α_s corrections for the mass as expected for the ratio of sum rules

    $Z_c(1^{++})$ can be pure heavy-light molecule states BUT NOT the $Y(1^{--})$

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- ♠ Extented to some other molecule states

  - $\bar{D}D(0^+),\ \bar{D}^*D^*(0^+),\ \bar{D}D_0^*(0^-),\ \bar{D}_1D_0^*(1^+),\ J/\psi S(1^-),\quad \ldots$

    Work in progress !

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- Inclusion of PT radiative corrections for Exotics
- Inclusion of NP contributions not induced by the quark propagator in external gluon fields: gluon condensates: more difficult !
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♠ Then, SVZ sum rules will continue to be useful & competitive !