Minimal Massive Gravity

or

The Third Way

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Based on

Minimal Massive 3D Gravity, E. Bergshoeff, O. Hohm, W. Merbis, A.J. Routh & PKT

Matter Coupling in 3D "Minimal Massive Gravity", A.S. Arvanitakis, A.J. Routh & PKT

FloratosFest, Athens, October 2014

A Question of Consistency

Consistency of Einstein equation $G_{\mu\nu} = T_{\mu\nu}$ requires

$$D_\mu T^{\mu\nu} = 0 \qquad\qquad (*)$$

1. T is a matter stress tensor: $(*)$ is a consequence of matter equations.

2. $(*)$ is an identity; e.g. $T_{\mu\nu} = -\Lambda g_{\mu\nu}$. More generally, this leads to higher-derivative modifications.

3. The third way.

3D Einstein gravity

- Action (parity preserving): $I_{\text{Einstein-Hilbert}}[g] + I_{\text{matter}}[\phi, g]$

- Equation is $G_{\mu\nu} = (8\pi G_3)\, T_{\mu\nu}$

- Matter determines curvature *uniquely*, so no gravitons

- In AdS vacuum of radius ℓ central charges $c_\pm$ of (putative) dual 2D CFT are [Brown& Henneaux]

$$c_\pm = \frac{3\ell}{2G_3}$$

[Valid in semi-classical limit for which $\ell/G_3 \gg 1$]

Topologically Massive Gravity

TMG (parity violating) action is $\frac{1}{\mu} I_{LCS}[g] + \sigma I_{\text{EH}}$ where μ is a mass parameter (and σ is a sign).

➸ Lorentz-Chern-Simons term is third-order in derivatives of g. Variation gives **Cotton** tensor ('curl' of **Schouten** tensor):

$$C_{\mu\nu} = \epsilon_\mu{}^{\rho\sigma} D_\rho S_{\sigma\nu} \,, \qquad \left(S_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{4} g_{\mu\nu} R\right)$$

So TMG equation is

$$\frac{1}{\mu} C_{\mu\nu} + \sigma G_{\mu\nu} = 0$$

➸ Propagates one spin-2 mode of mass μ, but positive energy requires "wrong-sign" EH term: $\sigma = -1$.

Bulk-Boundary clash for TMG

TMG has AdS vacuum ($\Lambda = -1/\ell^2$) and central charges of dual CFT are

$$c_\pm = \frac{3\ell}{2G_3}\left(\sigma \pm \frac{1}{\mu\ell}\right).$$

Positive c_+ *and* c_- requires $\sigma = 1$ but positive energy graviton (no ghost) requires $\sigma = -1$.

[$c_- = 0$ is special case, but resulting "chiral gravity" (Li, Song & Strominger) has non-unitary dual "log CFT".]

- Conclusion: TMG has no unitary quantum completion.

- Higher derivative "corrections" to TMG don't help.

The "third way"

Define $J^{\mu\nu} \equiv \frac{1}{2(\det g)} \varepsilon^{\mu\rho\sigma}\varepsilon^{\nu\tau\eta}S_{\rho\tau}S_{\sigma\eta}$

Satisfies identity $\sqrt{-\det g}\; D_\mu J^{\mu\nu} \equiv \varepsilon^{\nu\rho\sigma}S_\rho{}^\tau C_{\sigma\tau} \not\equiv 0.$

However, using source-free TMG equation, and symmetry of S,

$$\varepsilon^{\nu\rho\sigma}S_\rho{}^\tau C_{\sigma\tau} \;\propto\; \varepsilon^{\nu\rho\sigma}S_\rho{}^\tau G_{\sigma\tau} \;\equiv\; \varepsilon^{\nu\rho\sigma}S_\rho{}^\tau S_{\sigma\tau} \;\equiv\; 0$$

So J-tensor is conserved **as a consequence of TMG equation**.

➽ So far so good, but if we add J to the TMG equation then we change the equation used to establish $D_\mu J^{\mu\nu} = 0$.

Minimal Massive Gravity

Let's do it anyway! This gives source-free **MMG** equation

$$\frac{1}{\mu} C_{\mu\nu} + \sigma G_{\mu\nu} + \frac{\gamma}{\mu^2} J_{\mu\nu} = 0$$

for constant γ. Using this equation (and previous results for J-tensor) we now find that

$$D_\mu J^{\mu\nu} = \varepsilon^{\nu\rho\sigma} S_\rho{}^\tau J_{\sigma\tau} \equiv 0$$

Identity follows from 'ϵS^2' structure of J.

- MMG equation **is** consistent with Bianchi identities.
- Because $D_\mu J^{\mu\nu} \not\equiv 0$ there is no 'geometrical' action $I_{MMG}[g]$.

Matter coupling

Add source: $\frac{1}{\mu}C_{\mu\nu} + \sigma G_{\mu\nu} + \frac{\gamma}{\mu^2}J_{\mu\nu} = \mathcal{T}_{\mu\nu}$. Consistency requires

$$\sqrt{-\det g}\; D_\mu \mathcal{T}^{\mu\nu} = \frac{\gamma}{\mu}\; \varepsilon^{\nu\rho\sigma} S_\rho{}^\tau \mathcal{T}_{\sigma\tau}$$

For $\gamma = 0$ (TMG) we can take $\mathcal{T} = T$ (usual matter stress tensor).

➻ For $\gamma \neq 0$ (MMG) the source tensor is (!)

$$\begin{aligned} \mathcal{T}_{\mu\nu} &= \frac{1}{(1+\gamma\sigma)} T_{\mu\nu} + \frac{\gamma}{\mu(1+\gamma\sigma)^2}\; \epsilon_\mu{}^{\rho\sigma} D_\rho \hat{T}_{\sigma\nu} \\ &- \frac{\gamma^2}{(1+\gamma\sigma)^2\mu^2}\, \epsilon_\mu{}^{\rho\sigma}\epsilon_\nu{}^{\lambda\kappa} S_{\rho\lambda}\hat{T}_{\sigma\kappa} + \frac{\gamma^3}{2(1+\gamma\sigma)^4\mu^2}\, \epsilon_\mu{}^{\rho\sigma}\epsilon_\nu{}^{\lambda\kappa}\hat{T}_{\rho\lambda}\hat{T}_{\sigma\kappa} \end{aligned}$$

where $\hat{T}_{\mu\nu} = T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T$

The MMG action

Einstein-Cartan formulation of 3D GR involves Lorentz-vector 1-forms e^a (dreibein) and ω^a (Lorentz connection). For TMG we need additional Lagrange multiplier 1-form h^a for zero torsion constraint [Grumiller, Jackiw & Johansson; Carlip].

$$L_{TMG} = \sigma e \cdot R + h \cdot T + \frac{1}{\mu} L_{LCS}(\omega)$$

[We use 3D vector algebra, T is torsion and R curvature]

➽ For MMG we just add an extra term

$$L_{MMG} = L_{TMG} - \frac{\gamma}{2(1+\gamma\sigma)^2}\, e \cdot h \times h$$

A puzzle posed and solved

- Field equations allow (h, ω) to be eliminated algebraically, leaving a third-order equation for e. This is the MMG equation when expressed in terms of metric g.

- Puzzle: **why can't we eliminate (h, ω) in the *action* to get an action for e, and hence for g?** This can be done for TMG.

- Resolution: When $\gamma \neq 0$, elimination of (h, ω) requires use of e-eq. But then it is illegitimate to back-substitute into action.

- MMG goes the "third way".

Resolution of Bulk vs Boundary clash

- Linearization about AdS gives identical results to TMG

- Hamiltonian analysis of non-linear theory gives same phase-space dimension as TMG

- Characteristics of MMG equation same as TMG because J contains no '∂R' terms .

- But: we can choose MMG parameters such that graviton has positive energy **and** $c_\pm > 0$.

So MMG is viable candidate for semi-classical limit of a unitary quantum 3D massive gravity.

The last slide

Manolis.

Best wishes for the future!