APPENDIX I. REDUCED TRANSITION PROBABILITIES

The measured lifetime or cross section for a nuclear transition can be compared with a theoretical value to obtain a reduced transition probability, typically T_{exp}/T_{theory}, where $T=\frac{\ln 2}{t_{1/2}}$ is defined as the transition probability. These values can be used to help discern nuclear structure and transition properties.

1. Photon Transition Probabilities

For the kth photon of n photons deexciting a level, the partial half-life for photon emission $t_{1/2}(\gamma)$ is related to the level half-life $t_{1/2}$ by

$$t_{1/2}(\gamma) = t_{1/2}\sum_{i=1}^{n}(I_i^{\gamma})\frac{(1+\alpha_i)}{I_k} , \quad (1)$$

where the summation is over the intensity I_i^{γ} of all photons deexciting the level, corrected for the internal conversion coefficient α_i. For transitions with mixed multipolarities L and L+1, the photon half-life becomes

$$t_{1/2}(\gamma)^{L} = t_{1/2}(\gamma)\times(1+\delta^2) , \quad (2)$$

$$t_{1/2}(\gamma)^{L+1} = t_{1/2}(\gamma)\times\frac{(1+\delta^2)}{\delta^2} , \quad (3)$$

where δ is the mixing ratio. For electric 2^L (EL) and magnetic 2^L (ML) transitions, the experimental photon transition probability can be compared with theoretical values by the equations

$$T_{\gamma}^{EL} = \frac{8\pi(L+1)e^2b^L}{L[(2L+1)!!]^2\hbar}\left(\frac{E_\gamma}{\hbar c}\right)^{2L+1} B(EL)\downarrow , \quad (4)$$

and

$$T_{\gamma}^{ML} = \frac{8\pi(L+1)\mu_N^2b^{L-1}}{L[(2L+1)!!]^2\hbar}\left(\frac{E_\gamma}{\hbar c}\right)^{2L+1} B(ML)\downarrow , \quad (5)$$

respectively. The constants in equations (2) and (3) are $\hbar c = 197.327\times10^{-10}\ keV\ cm$, $\hbar = 6.58212\times10^{-19}\ keV\ s$, $e^2 = 1.440\times10^{-10}\ keV\ cm$, $\mu_N^2 = 1.5922\times10^{-38}\ keV\ cm^3$; and $b = 10^{-24}\ cm^2$. B(EL)↓ and B(ML)↓ are the transition nuclear matrix elements which may be calculated with a theoretical model for comparison with experiment. Weisskopf[1] derived the following single particle estimates for these matrix elements based on the shell model.

$$B_{SP}(EL) = \frac{1}{4\pi b^L}\left(\frac{3}{3+L}\right)^2 R^{2L} \quad (6)$$

$$B_{SP}(ML) = \frac{10}{\pi b^{L-1}}\left(\frac{3}{3+L}\right)^2 R^{2L-2} \quad (7)$$

Here, $R = 1.2\times10^{-13}A^{1/3}cm$. The single particle photon half-life can be calculated for electric transitions (equations (2) and (4)) and magnetic transitions (equations (3) and (5)) with

$$t_{1/2}(\gamma)(EL)_{SP} = \frac{ln2\ L[(2L+1)!!]^2\hbar}{2(L+1)e^2R^{2L}}\left(\frac{3+L}{3}\right)^2\left(\frac{\hbar c}{E_\gamma}\right)^{2L+1} \quad (8)$$

and

$$t_{1/2}(\gamma)(ML)_{SP} = \frac{ln2\ L[(2L+1)!!]^2\hbar}{80(L+1)\mu_N^2R^{2L-2}}\left(\frac{3+L}{3}\right)^2\left(\frac{\hbar c}{E_\gamma}\right)^{2L+1} . \quad (9)$$

[1] J.M. Blatt and V.F. Weisskopf, *Theoretical Nuclear Physics*, John Wiley and Sons, New York (1952), p. 627.

The Weisskopf single-particle transition probability is defined by

$$B(EL;ML)(W.u.) = \frac{t^{\gamma}_{1/2}(EL;ML)_{SP}}{t^{\gamma}_{1/2}(EL;ML)_{\exp}} . \qquad (10)$$

Simple equations for single-particle half-lives of L=1-5 order transitions are given in Table 1. Single-particle transition half-lives, including the contribution from internal conversion, are plotted in Figure 1 (electric transitions) and Figure 2 (magnetic transitions).

Table 1. Formulae for single-particle transition half-lives, corrected for internal conversion.

Electric	$t^{\gamma}_{1/2}$(s)	Magnetic	$t^{\gamma}_{1/2}$(s)
E1	$\frac{6.76\times10^{-6}}{E_\gamma^3 A^{2/3}}$	M1	$\frac{2.20\times10^{-5}}{E_\gamma^3}$
E2	$\frac{9.52\times10^{6}}{E_\gamma^5 A^{4/3}}$	M2	$\frac{3.10\times10^{7}}{E_\gamma^5 A^{2/3}}$
E3	$\frac{2.04\times10^{19}}{E_\gamma^7 A^{2}}$	M3	$\frac{6.66\times10^{19}}{E_\gamma^7 A^{4/3}}$
E4	$\frac{6.50\times10^{31}}{E_\gamma^9 A^{8/3}}$	M4	$\frac{2.12\times10^{32}}{E_\gamma^9 A^{2}}$
E5	$\frac{2.89\times10^{44}}{E_\gamma^{11} A^{10/3}}$	M5	$\frac{9.42\times10^{44}}{E_\gamma^{11} A^{8/3}}$

Systematics for the ratio of the single-particle half-life to the experimental half-life can be used to help restrict the range of possible transition multipolarities. Recommended upper limits for this ratio have been derived by Endt for 5≤A≤44[2], 45≤A≤90[3], 91≤A≤150[4], and by Martin for A>150[5]. These limits are summarized in Table 2.

Table 2. $t_{1/2}$(W.u.)/$t_{1/2}$(exp) (Recommended Upper Limits)

Multipolarity	5≤A≤44[a]	45≤A≤90	91≤A≤150	A≥151
E1 (isovector)	0.5[b]	0.01	0.01	0.01
E2 (isoscalar)	100	300	300	1000
E3	50	100	100	100
E4	50	100	30	
M1 (isovector)	10[c]	3	3	2
M2 (isovector)	5	1	1	1
M3 (isovector)	10	10	10	10
M4		30	30	10

[a] 0.002 for E1 (isoscalar), 5 for E2 (isovector), 0.05 for M1 (isoscalar), and 0.2 for M2 (isoscalar).

[b] 0.1 for 21≤A≤44

[c] 5 for 21≤A≤44

E2 transitions may be collectively enhanced in deformed nuclei. The recommended upper limits reflect this effect in normally deformed nuclear bands. Recent experiments measuring superdeformed nuclear bands suggest that these limits should be raised by an order-of-magnitude for nuclei with large deformations.

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[2]P.M. Endt, *At. Data Nucl. Data Tables* **55**, 171 (1993).

[3]P.M. Endt, *At. Data Nucl. Data Tables* **23**, 547 (1979).

[4]P.M. Endt, *At. Data Nucl. Data Tables* **26**, 47 (1981).

[5]M.J. Martin, *Nucl. Data Sheets* **74**, ix (1995).