INTRODUCTION OF EFFECTIVE CHARGES IN THE VECTOR BOSON MODEL

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EFFECTIVE CHARGES

- VBM intra- and interband B(E2) transition values depart from those in the favored SU(3)-irrep (λ_0, μ_0);
- After the determination of (λ_0, μ_0) the VBM calculations are repeated for various SU(3)-irreps by scaling the theoretical B(E2) values with effective charges.

$$e^2_{\text{eff-intra}} = \left(\frac{\lambda_0 + \mu_0 + 1}{\lambda + \mu + 1}\right)^2 , \quad \text{for intraband transitions ;}$$

$$e^2_{\text{eff-inter}} = \left(\frac{\mu_0 + 1}{\mu + 1}\right)^2 , \quad \text{for interband transitions .}$$

Alternative formula for interband transitions ⟶

$$\boxed{e^2_{\text{eff-intra}} = \left(\frac{\lambda_0 + \mu_0}{\lambda + \mu}\right)^2}$$

No new parameters are introduced into the model!

Ref: **Vector boson model application with SU(3)-irrep dependent effective charges, Minkov, N., Grigorova, D., & Bonatsos, D. (2025), Nuclear Theory (Vol. 42), p. p. 111-121**

ALGORITHM AND PRECISION EVALUATION

- Step 1: Calculations without effective charges → determination of (λ_0, μ_0);
- Step 2: Inclusion of the (λ_0, μ_0)-values into the effective charge expressions and (re-)calculation of the energy spectrum and transition probabilities for the irrep(s) of interest with the effective charges.

$$\sigma_E = \sqrt{\frac{1}{n_E}\sum_{\nu=1}^{n_E}\left[E_\nu^{\mathrm{th}} - E_\nu^{\mathrm{exp}}\right]^2}, \quad \sigma_B = \sqrt{\frac{1}{n_B}\sum_{\nu=1}^{n_B}\left[B(E2)_\nu^{\mathrm{th}} - B(E2)_\nu^{\mathrm{exp}}\right]^2}$$

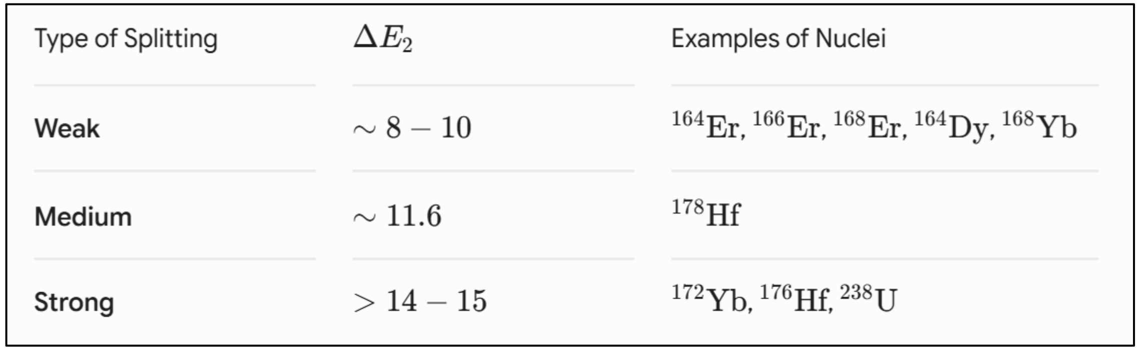
$$\boxed{\sigma_T = \sigma_E + \sigma_B}$$

Ref: **Vector boson model application with SU(3)-irrep dependent effective charges, Minkov, N., Grigorova, D., & Bonatsos, D. (2025), Nuclear Theory (Vol. 42), p. 111-121**

ENERGY SPLITTING

The splitting ΔE_2 is the relative energy difference between the first excited states of the ground g- and γ-bands, which measures how strongly the g- and γ-band are coupled with respect to the VBM framework.

$$\Delta E_2 = \frac{E_{2^+}^{(g)} - E_{2^+}^{(\gamma)}}{E_{2^+}^{(g)}}$$

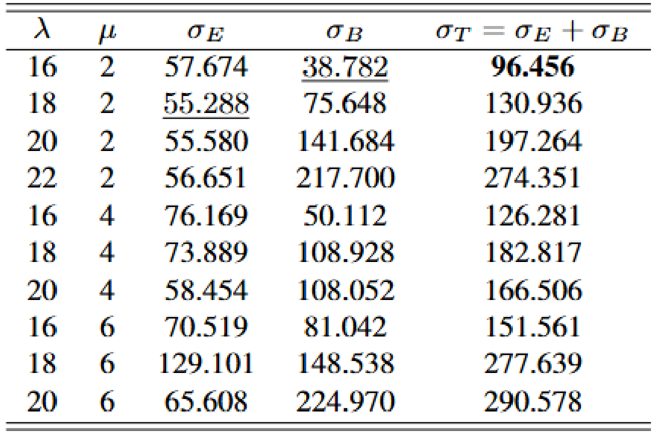


Type of Splitting	ΔE_2	Examples of Nuclei
Weak	$\sim 8 - 10$	$^{164}\mathrm{Er}$, $^{166}\mathrm{Er}$, $^{168}\mathrm{Er}$, $^{164}\mathrm{Dy}$, $^{168}\mathrm{Yb}$
Medium	~ 11.6	$^{178}\mathrm{Hf}$
Strong	$> 14 - 15$	$^{172}\mathrm{Yb}$, $^{176}\mathrm{Hf}$, $^{238}\mathrm{U}$

Ref: Minkov, N., Drenska, S. B., Raychev, P. P., Roussev, R. P., & Bonatsos, D. (1997). Broken SU(3) symmetry in deformed even-even nuclei. *Physical Review C*, *55*(5), 2345-2360

NUCLEI WITH WEAK ENERGY SPLITTING – ^{166}ER AND ^{168}ER

NUCLEI WITH WEAK ENERGY SPLITTING – ^{166}ER



λ	μ	σ_E	σ_B	$\sigma_T = \sigma_E + \sigma_B$
16	2	57.674	38.782	**96.456**
18	2	55.288	75.648	130.936
20	2	55.580	141.684	197.264
22	2	56.651	217.700	274.351
16	4	76.169	50.112	126.281
18	4	73.889	108.928	182.817
20	4	58.454	108.052	166.506
16	6	70.519	81.042	151.561
18	6	129.101	148.538	277.639
20	6	65.608	224.970	290.578

Table 3: Energy and B(E2) RMS deviations σ_E (in keV) and σ_B (in W.u.) and their sum σ_T (in relative units, r.u.) obtained for various (λ, μ)-values in ^{166}Er without use of B(E2) scaling factors. The lowest σ_E and σ_B values are underlined and that for σ_T is bolded.

$(\lambda_0, \mu_0) = (16, 2)$
$(\lambda_f, \mu_f) = (20, 4)$

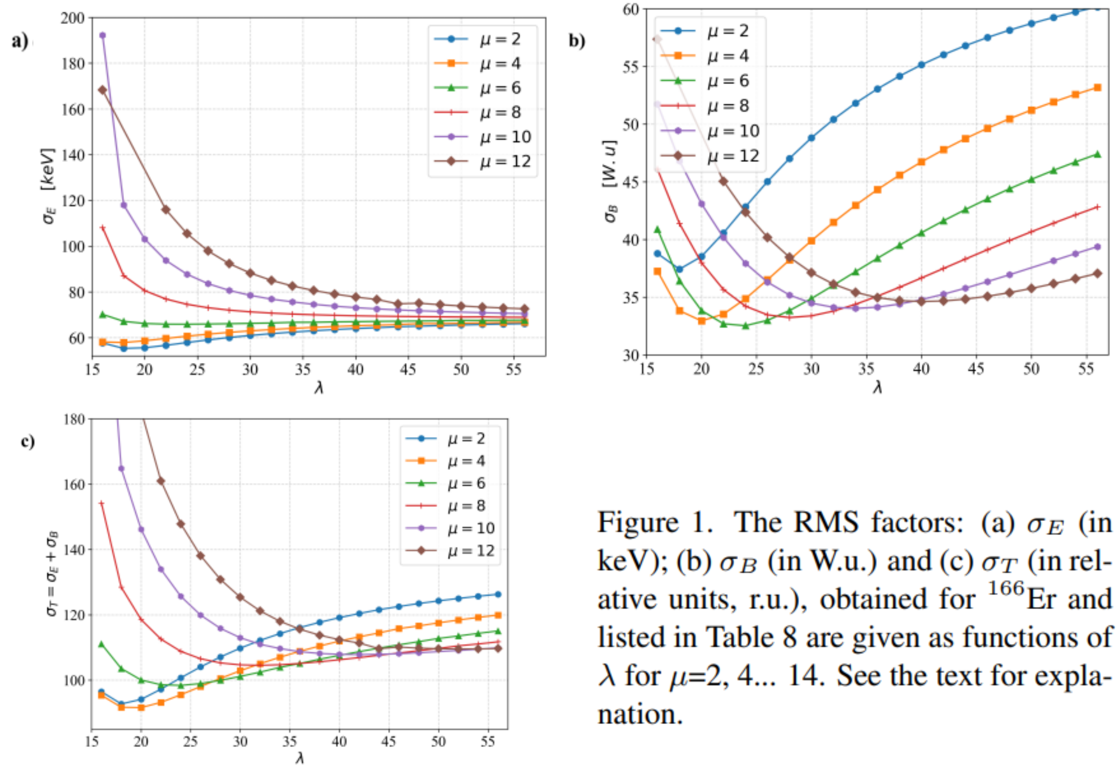
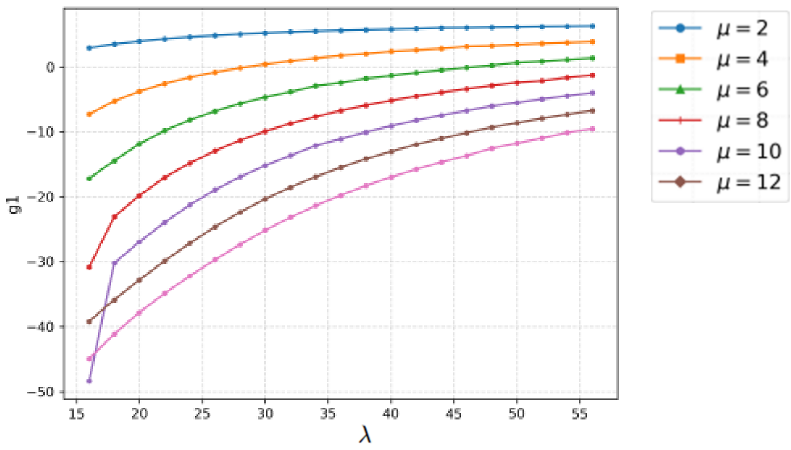
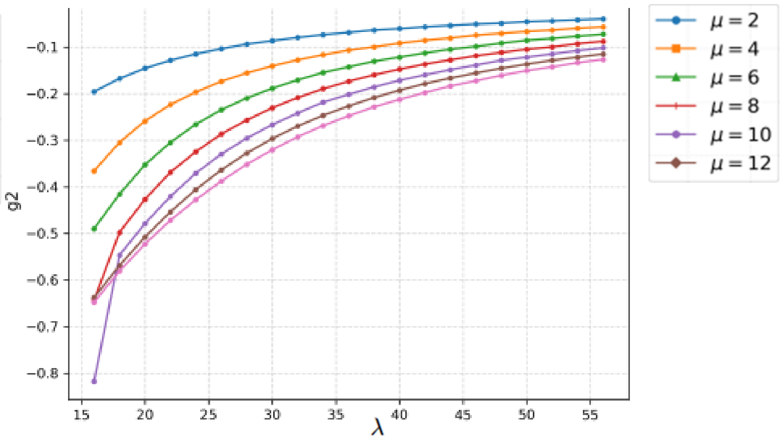


Figure 1. The RMS factors: (a) σ_E (in keV); (b) σ_B (in W.u.) and (c) σ_T (in relative units, r.u.), obtained for ^{166}Er and listed in Table 8 are given as functions of λ for μ=2, 4... 14. See the text for explanation.

HAMILTONIAN PARAMETERS FOR ^{166}ER





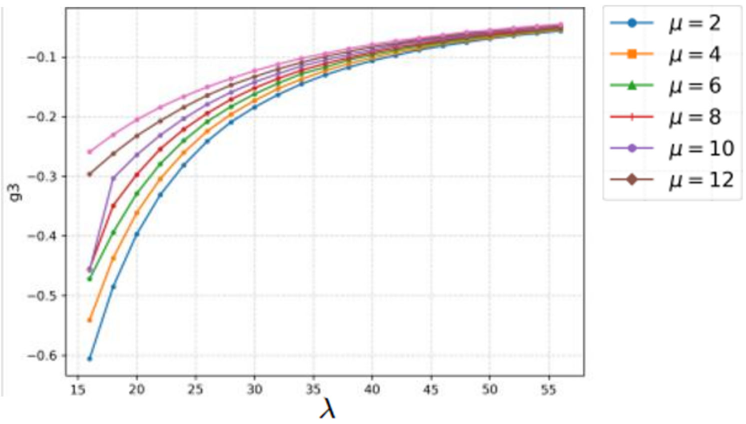


Figure 3. Optimization parameters: (a) g_1; (b) g_2 and (c) g_3, obtained for ^{166}Er and listed in Table 8 are given as functions of λ for μ=2, 4... 14. See the text for explanation.

COMAPRISON WITH PAST RESEARCH – ^{166}ER

PHYSICAL REVIEW C VOLUME 55, NUMBER 5 MAY 1997

Broken SU(3) symmetry in deformed even-even nuclei

N. Minkov,[1,*] S. B. Drenska,[1,†] P. P. Raychev,[1,2,‡] R. P. Roussev,[1,§] and Dennis Bonatsos[3,‖]
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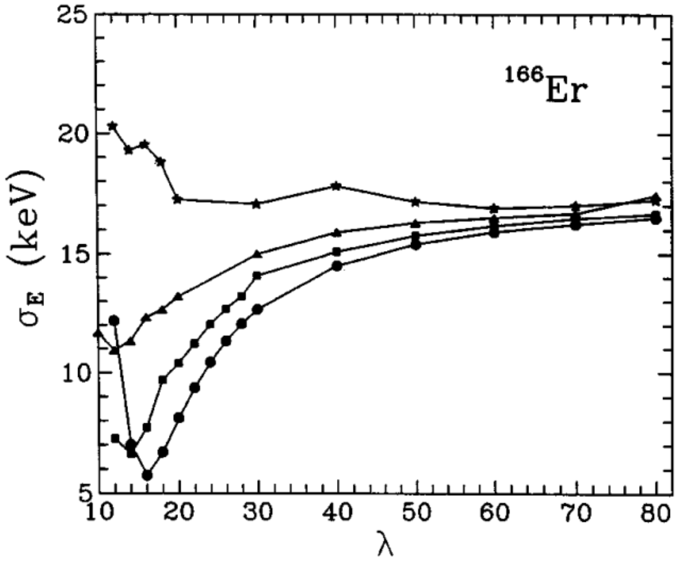
[2]*Dipartimento di Scienze Fisiche, Università di Napoli "Federico II," Mostra d' Oltremare Pad. 19, I-80185, Napoli, Italy*
[3]*Institute of Nuclear Physics, N.C.S.R. "Demokritos," GR-15310 Aghia Paraskevi, Attiki, Greece*



● → μ=2
■ → μ=4
▲ → μ=6
* → μ=8

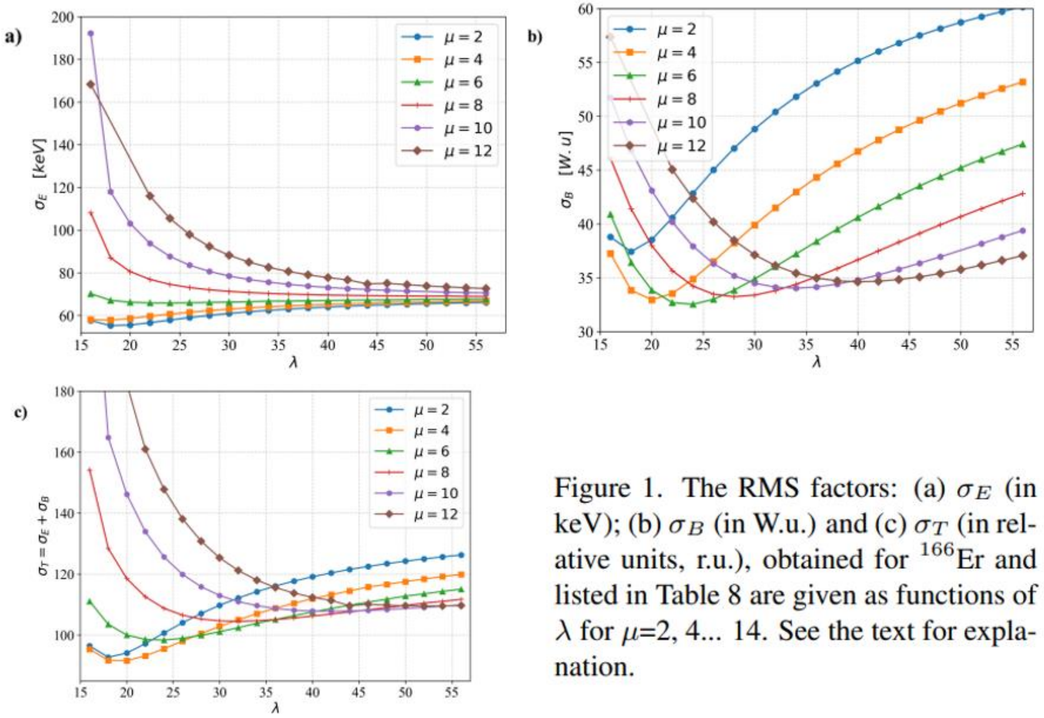
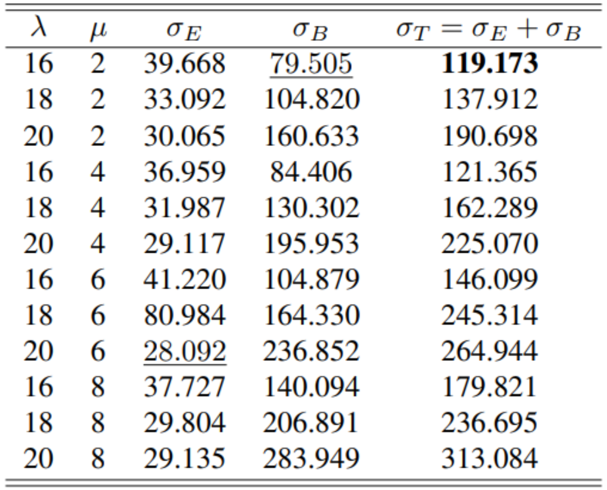


Figure 1. The RMS factors: (a) σ_E (in keV); (b) σ_B (in W.u.) and (c) σ_T (in relative units, r.u.), obtained for ^{166}Er and listed in Table 8 are given as functions of λ for μ=2, 4... 14. See the text for explanation.

- PRC97: (λ, μ) = (16, 2)

Current results: $(\lambda_f, \mu_f) = (20, 4)$

^{168}ER – WEAK ENERGY SPLITTING



λ	μ	σ_E	σ_B	$\sigma_T = \sigma_E + \sigma_B$
16	2	39.668	79.505	**119.173**
18	2	33.092	104.820	137.912
20	2	30.065	160.633	190.698
16	4	36.959	84.406	121.365
18	4	31.987	130.302	162.289
20	4	29.117	195.953	225.070
16	6	41.220	104.879	146.099
18	6	80.984	164.330	245.314
20	6	28.092	236.852	264.944
16	8	37.727	140.094	179.821
18	8	29.804	206.891	236.695
20	8	29.135	283.949	313.084

Table 5. Energy and B(E2) RMS deviations σ_E (in keV) and σ_B (in W.u.) and their sum σ_T (in relative units, r.u.) calculated for various (λ, μ)-values in ^{168}Er without use of scaling factors in the B(E2)s. The lowest σ_E- and σ_B- values are underlined and that for σ_T is bolded.

- $(\lambda_0, \mu_0) = (16, 2)$
- $(\lambda_f, \mu_f) = (22, 8)$

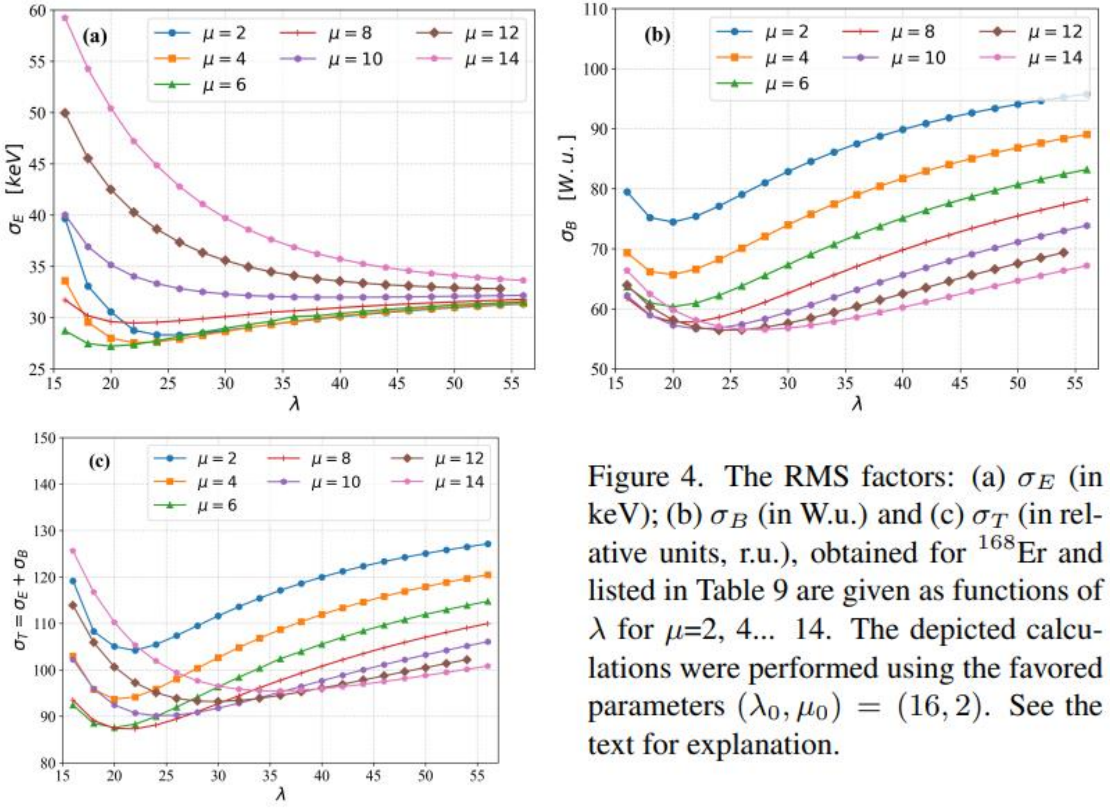


Figure 4. The RMS factors: (a) σ_E (in keV); (b) σ_B (in W.u.) and (c) σ_T (in relative units, r.u.), obtained for ^{168}Er and listed in Table 9 are given as functions of λ for μ=2, 4... 14. The depicted calculations were performed using the favored parameters $(\lambda_0, \mu_0) = (16, 2)$. See the text for explanation.

COMPARISON WITH PAST RESEARCH – ^{168}ER

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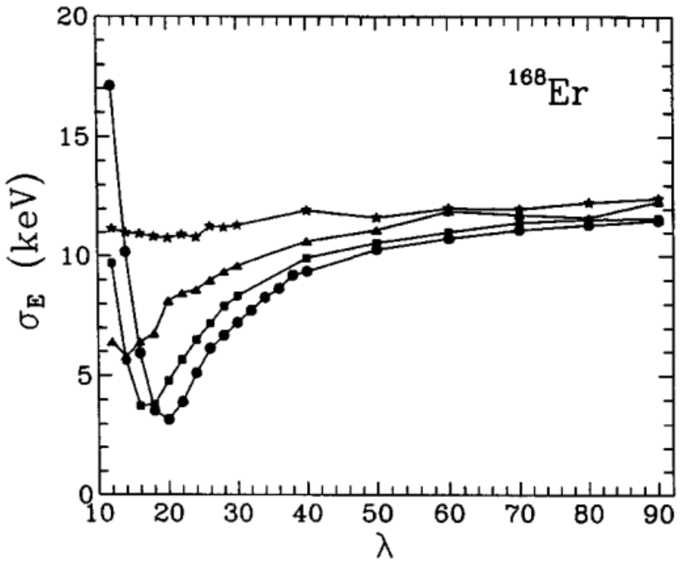
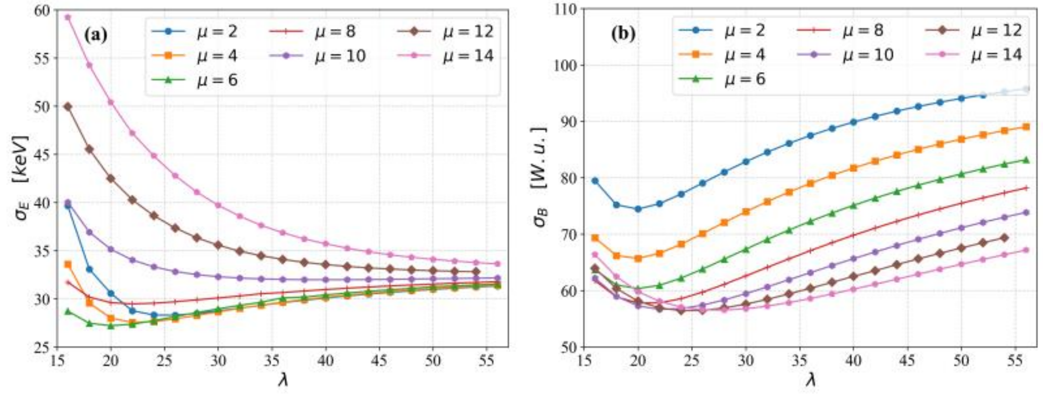


● → μ=2

■ → μ=4

▲ → μ=6

* → μ=8



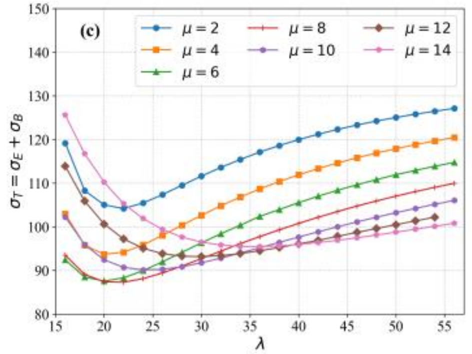


Figure 4. The RMS factors: (a) σ_E (in keV); (b) σ_B (in W.u.) and (c) σ_T (in relative units, r.u.), obtained for ^{168}Er and listed in Table 9 are given as functions of λ for μ=2, 4... 14. The depicted calculations were performed using the favored parameters $(\lambda_0, \mu_0) = (16, 2)$. See the text for explanation.

- PRC97: (λ, μ) = (20, 2)

Current results: (λ_f, μ_f) = (22, 8)

COMPARISON WITH PAST RESEARCH – ^{168}ER. BEHAVIOR OF THE HAMILTONIAN PARAMETERS

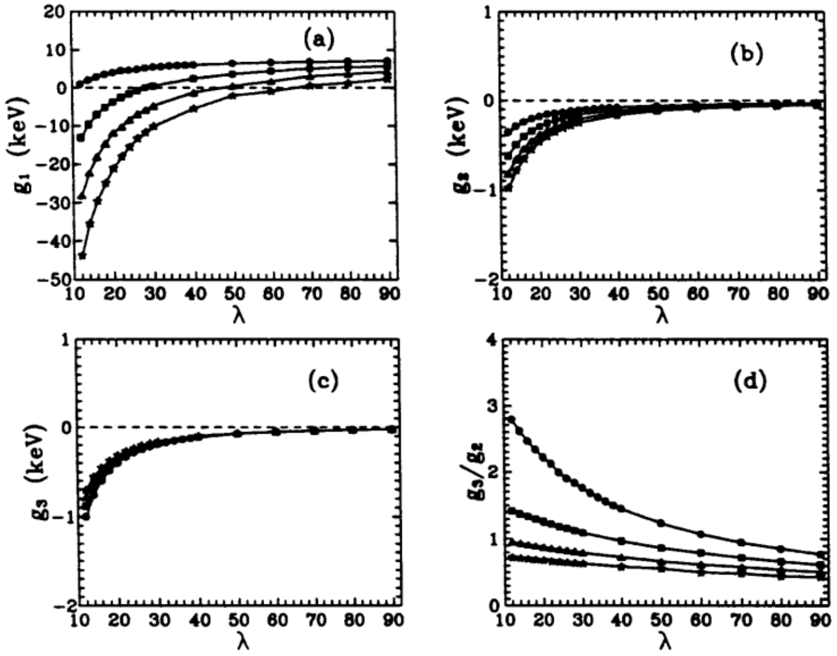


FIG. 2. The Hamiltonian parameters g_1,g_2,g_3 [Eq. (5)] and the ratio g_3/g_2, adjusted for the nucleus ^{168}Er, are plotted [in (a), (b), (c), and (d), respectively] as functions of the quantum number λ at μ=2 (circlets), μ=4 (squares), μ=6 (triangles), and μ=8 (asterisks).

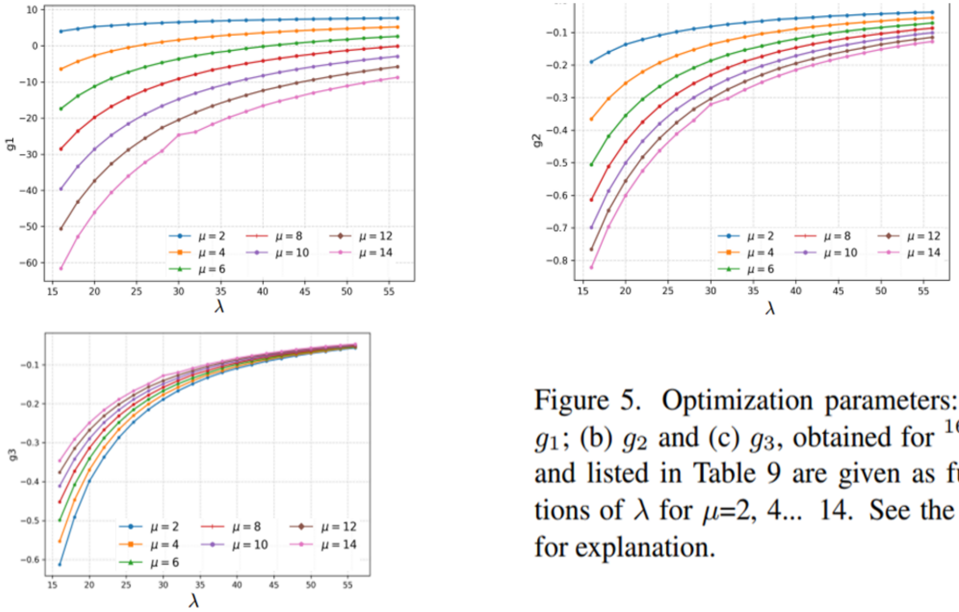


Figure 5. Optimization parameters: (a) g_1; (b) g_2 and (c) g_3, obtained for ^{168}Er and listed in Table 9 are given as functions of λ for μ=2, 4... 14. See the text for explanation.

^{166}ER AND ^{168}ER – COMPARISON WITH PROXY SU(3)

Favored irreps ^{166}Er:

Current results:

$(\lambda_f, \mu_f) = (20, 4)$

$\lambda/\mu = 5$

Proxy SU(3):

$(\lambda, \mu) = (52, 14)$

$\lambda/\mu = 3.7$

Favored irreps ^{168}Er:

Current results:

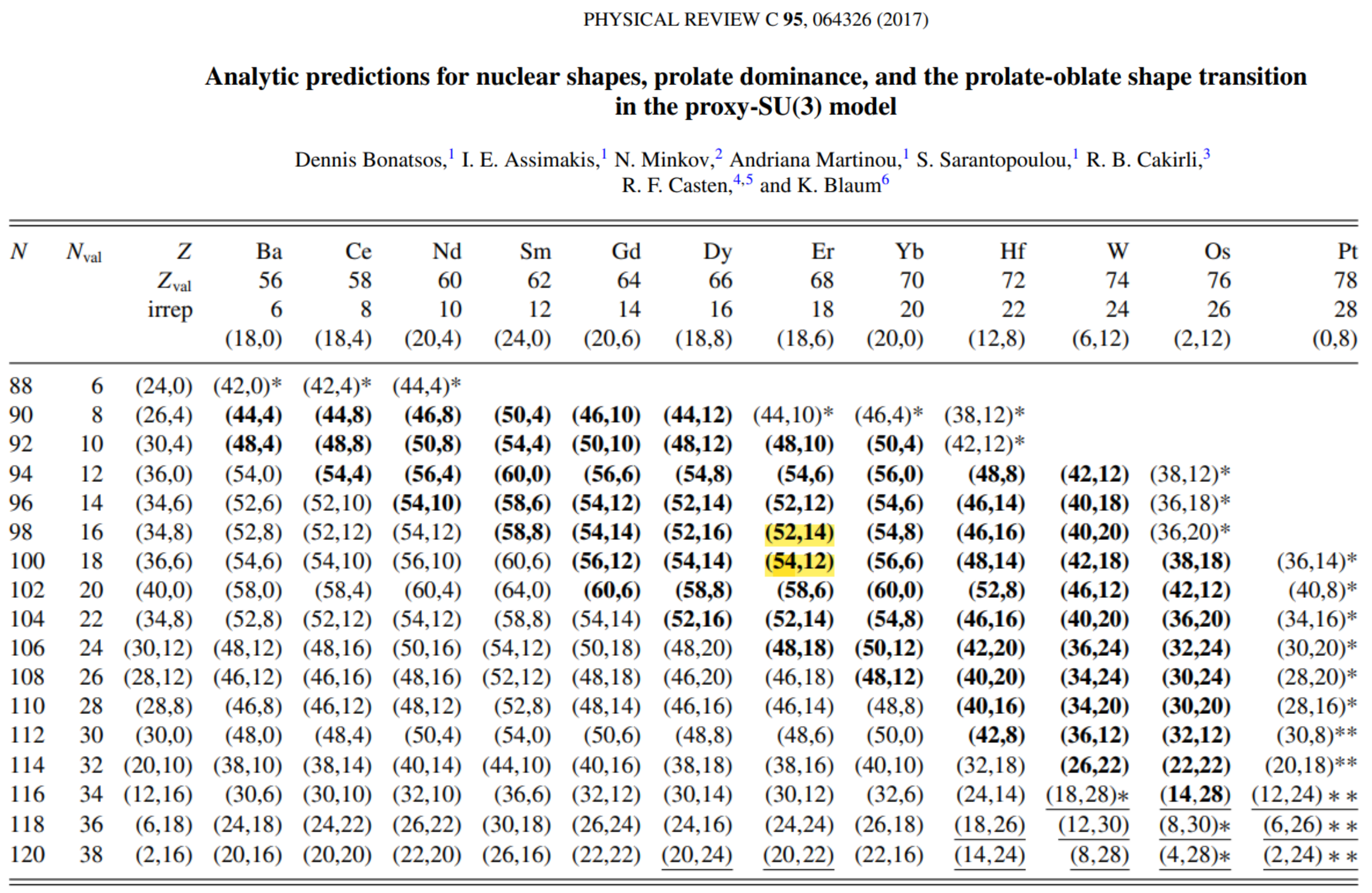
$(\lambda_f, \mu_f) = (22, 8)$

$\lambda/\mu = 2.75$

Proxy SU(3):

$(\lambda, \mu) = (54, 12)$

$\lambda/\mu = 4.5$



PHYSICAL REVIEW C **95**, 064326 (2017)

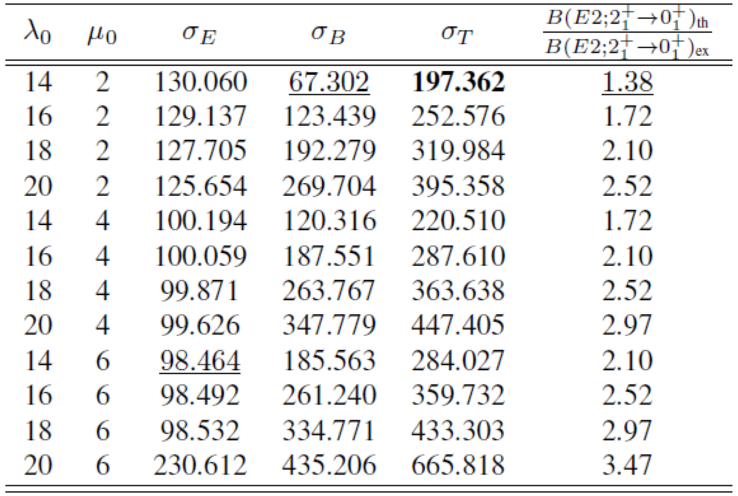
Analytic predictions for nuclear shapes, prolate dominance, and the prolate-oblate shape transition in the proxy-SU(3) model

Dennis Bonatsos,[1] I. E. Assimakis,[1] N. Minkov,[2] Andriana Martinou,[1] S. Sarantopoulou,[1] R. B. Cakirli,[3]
R. F. Casten,[4,5] and K. Blaum[6]

N	N_{val}	Z	Ba	Ce	Nd	Sm	Gd	Dy	Er	Yb	Hf	W	Os	Pt
		Z_{val}	56	58	60	62	64	66	68	70	72	74	76	78
		irrep	6	8	10	12	14	16	18	20	22	24	26	28
			(18,0)	(18,4)	(20,4)	(24,0)	(20,6)	(18,8)	(18,6)	(20,0)	(12,8)	(6,12)	(2,12)	(0,8)
88	6	(24,0)	(42,0)*	(42,4)*	(44,4)*									
90	8	(26,4)	**(44,4)**	**(44,8)**	**(46,8)**	**(50,4)**	**(46,10)**	**(44,12)**	(44,10)*	(46,4)*	(38,12)*			
92	10	(30,4)	**(48,4)**	**(48,8)**	**(50,8)**	**(54,4)**	**(50,10)**	**(48,12)**	**(48,10)**	**(50,4)**	(42,12)*			
94	12	(36,0)	(54,0)	**(54,4)**	**(56,4)**	**(60,0)**	**(56,6)**	**(54,8)**	**(54,6)**	**(56,0)**	**(48,8)**	**(42,12)**	(38,12)*	
96	14	(34,6)	(52,6)	(52,10)	**(54,10)**	**(58,6)**	**(54,12)**	**(52,14)**	**(52,12)**	**(54,6)**	**(46,14)**	**(40,18)**	(36,18)*	
98	16	(34,8)	(52,8)	(52,12)	(54,12)	**(58,8)**	**(54,14)**	**(52,16)**	**(52,14)**	**(54,8)**	**(46,16)**	**(40,20)**	(36,20)*	
100	18	(36,6)	(54,6)	(54,10)	(56,10)	(60,6)	**(56,12)**	**(54,14)**	**(54,12)**	**(56,6)**	**(48,14)**	**(42,18)**	**(38,18)**	(36,14)*
102	20	(40,0)	(58,0)	(58,4)	(60,4)	(64,0)	**(60,6)**	**(58,8)**	**(58,6)**	**(60,0)**	**(52,8)**	**(46,12)**	**(42,12)**	(40,8)*
104	22	(34,8)	(52,8)	(52,12)	(54,12)	(58,8)	(54,14)	**(52,16)**	**(52,14)**	**(54,8)**	**(46,16)**	**(40,20)**	**(36,20)**	(34,16)*
106	24	(30,12)	(48,12)	(48,16)	(50,16)	(54,12)	(50,18)	(48,20)	**(48,18)**	**(50,12)**	**(42,20)**	**(36,24)**	**(32,24)**	(30,20)*
108	26	(28,12)	(46,12)	(46,16)	(48,16)	(52,12)	(48,18)	(46,20)	(46,18)	**(48,12)**	**(40,20)**	**(34,24)**	**(30,24)**	(28,20)*
110	28	(28,8)	(46,8)	(46,12)	(48,12)	(52,8)	(48,14)	(46,16)	(46,14)	(48,8)	**(40,16)**	**(34,20)**	**(30,20)**	(28,16)*
112	30	(30,0)	(48,0)	(48,4)	(50,4)	(54,0)	(50,6)	(48,8)	(48,6)	(50,0)	**(42,8)**	**(36,12)**	**(32,12)**	(30,8)**
114	32	(20,10)	(38,10)	(38,14)	(40,14)	(44,10)	(40,16)	(38,18)	(38,16)	(40,10)	(32,18)	**(26,22)**	**(22,22)**	(20,18)**
116	34	(12,16)	(30,6)	(30,10)	(32,10)	(36,6)	(32,12)	(30,14)	(30,12)	(32,6)	(24,14)	(18,28)*	**(14,28)**	(12,24) * *
118	36	(6,18)	(24,18)	(24,22)	(26,22)	(30,18)	(26,24)	(24,16)	(24,24)	(26,18)	(18,26)	(12,30)	(8,30)*	(6,26) * *
120	38	(2,16)	(20,16)	(20,20)	(22,20)	(26,16)	(22,22)	(20,24)	(20,22)	(22,16)	(14,24)	(8,28)	(4,28)*	(2,24) * *

NUCLEI WITH INTERMEDIATE AND STRONG ENERGY SPLITTING – ^{178}HF AND ^{238}U

^{178}HF – INTERMEDIATE ENERGY SPLITTING



λ_0	μ_0	σ_E	σ_B	σ_T	$\frac{B(E2;2_1^+\to 0_1^+)_{th}}{B(E2;2_1^+\to 0_1^+)_{ex}}$
14	2	130.060	$\underline{67.302}$	**197.362**	$\underline{1.38}$
16	2	129.137	123.439	252.576	1.72
18	2	127.705	192.279	319.984	2.10
20	2	125.654	269.704	395.358	2.52
14	4	100.194	120.316	220.510	1.72
16	4	100.059	187.551	287.610	2.10
18	4	99.871	263.767	363.638	2.52
20	4	99.626	347.779	447.405	2.97
14	6	$\underline{98.464}$	185.563	284.027	2.10
16	6	98.492	261.240	359.732	2.52
18	6	98.532	334.771	433.303	2.97
20	6	230.612	435.206	665.818	3.47

Table 6. Energy and B(E2) RMS deviations σ_E (in keV) and σ_B (in W.u.) and their sum σ_T (in relative units, r.u.) calculated for various (λ, μ)-values in ^{178}Hf after the completion of Step (ii') ref to the modified algorithm. The lowest σ_E - and σ_B - values are underlined and that for σ_T is bolded.

$(\lambda_0, \mu_0) = (14, 2)$

$(\lambda_f, \mu_f) = (46, 20)$

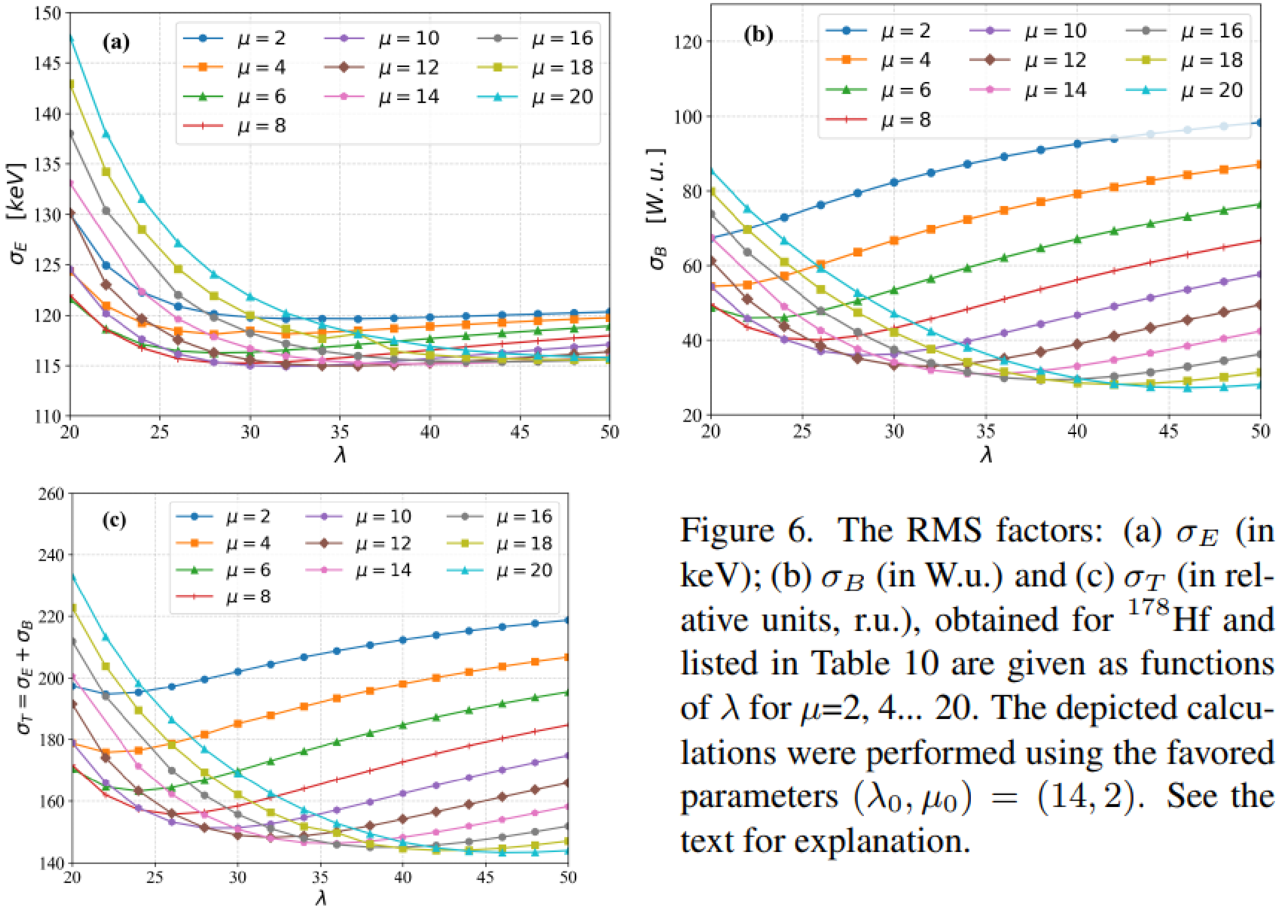
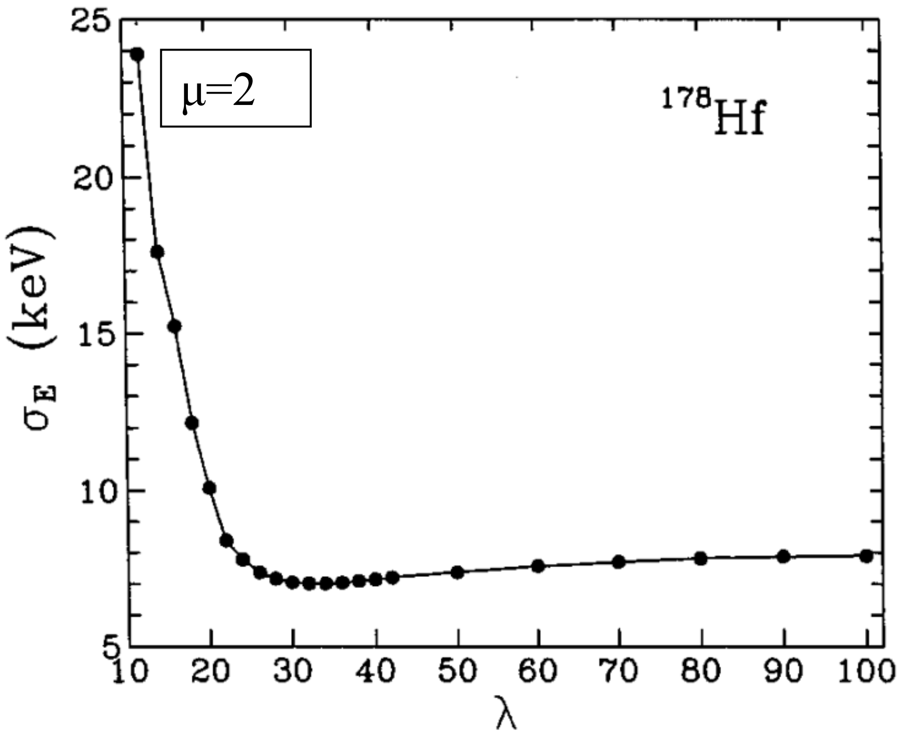


Figure 6. The RMS factors: (a) σ_E (in keV); (b) σ_B (in W.u.) and (c) σ_T (in relative units, r.u.), obtained for ^{178}Hf and listed in Table 10 are given as functions of λ for μ=2, 4... 20. The depicted calculations were performed using the favored parameters $(\lambda_0, \mu_0) = (14, 2)$. See the text for explanation.

COMPARISON WITH PAST RESEARCH – ^{178}HF



- PRC97: (λ, μ) = (30-36, 2)

Current results: (λ_f, μ_f) = (46, 20)

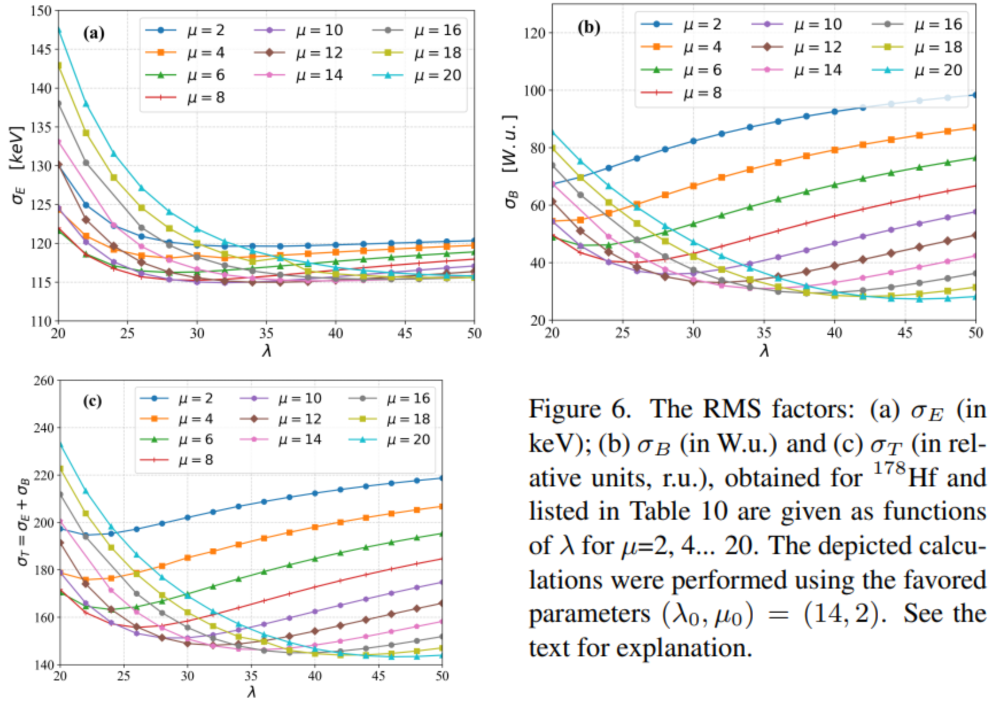


Figure 6. The RMS factors: (a) σ_E (in keV); (b) σ_B (in W.u.) and (c) σ_T (in relative units, r.u.), obtained for ^{178}Hf and listed in Table 10 are given as functions of λ for μ=2, 4... 20. The depicted calculations were performed using the favored parameters $(\lambda_0, \mu_0) = (14, 2)$. See the text for explanation.

Note:

- Good match with the RMS plot for the energy!
- New favored irrep due to modifications of the algorithm.

COMPARISON WITH PROXY SU(3) – ^{178}HF

Favored irreps ^{178}Hf:

Current results:

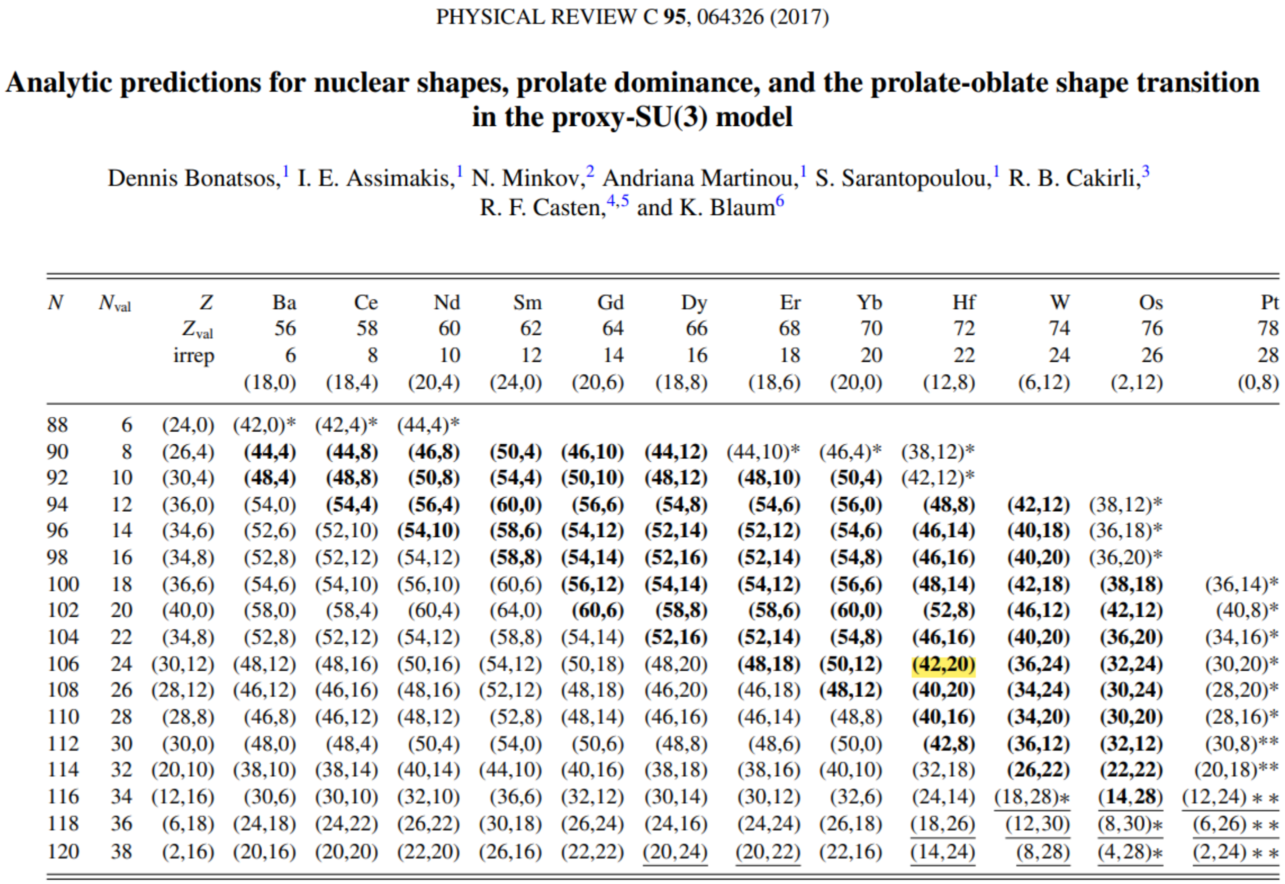
$(\lambda_f, \mu_f) = (46, 20)$

$\lambda/\mu = 2.3$

Proxy SU(3):

$(\lambda, \mu) = (42, 20)$

$\lambda/\mu = 2.1$



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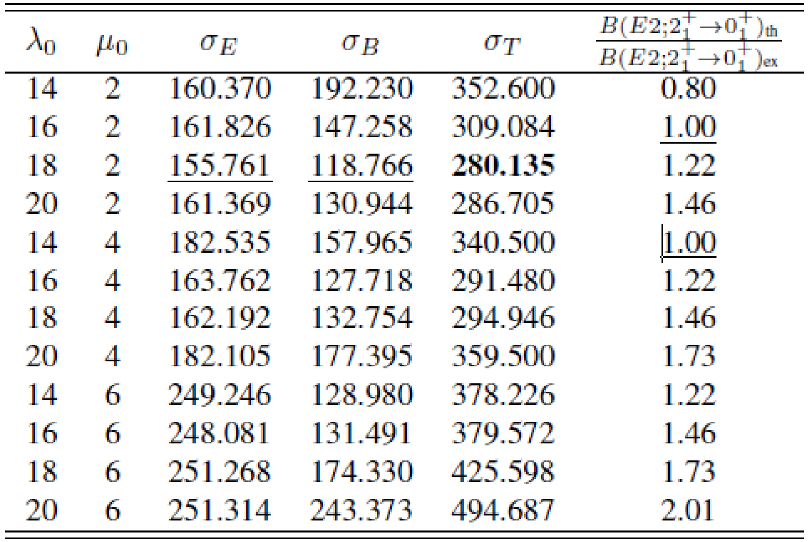
R. F. Casten,[4,5] and K. Blaum[6]

N	N_{val}	Z	Ba	Ce	Nd	Sm	Gd	Dy	Er	Yb	Hf	W	Os	Pt
		Z_{val}	56	58	60	62	64	66	68	70	72	74	76	78
		irrep	6	8	10	12	14	16	18	20	22	24	26	28
			(18,0)	(18,4)	(20,4)	(24,0)	(20,6)	(18,8)	(18,6)	(20,0)	(12,8)	(6,12)	(2,12)	(0,8)
88	6	(24,0)	(42,0)*	(42,4)*	(44,4)*									
90	8	(26,4)	**(44,4)**	**(44,8)**	**(46,8)**	**(50,4)**	**(46,10)**	**(44,12)**	(44,10)*	(46,4)*	(38,12)*			
92	10	(30,4)	**(48,4)**	**(48,8)**	**(50,8)**	**(54,4)**	**(50,10)**	**(48,12)**	**(48,10)**	**(50,4)**	(42,12)*			
94	12	(36,0)	(54,0)	**(54,4)**	**(56,4)**	**(60,0)**	**(56,6)**	**(54,8)**	**(54,6)**	**(56,0)**	**(48,8)**	**(42,12)**	(38,12)*	
96	14	(34,6)	(52,6)	(52,10)	**(54,10)**	**(58,6)**	**(54,12)**	**(52,14)**	**(52,12)**	**(54,6)**	**(46,14)**	**(40,18)**	(36,18)*	
98	16	(34,8)	(52,8)	(52,12)	(54,12)	**(58,8)**	**(54,14)**	**(52,16)**	**(52,14)**	**(54,8)**	**(46,16)**	**(40,20)**	(36,20)*	
100	18	(36,6)	(54,6)	(54,10)	(56,10)	(60,6)	**(56,12)**	**(54,14)**	**(54,12)**	**(56,6)**	**(48,14)**	**(42,18)**	**(38,18)**	(36,14)*
102	20	(40,0)	(58,0)	(58,4)	(60,4)	(64,0)	**(60,6)**	**(58,8)**	**(58,6)**	**(60,0)**	**(52,8)**	**(46,12)**	**(42,12)**	(40,8)*
104	22	(34,8)	(52,8)	(52,12)	(54,12)	(58,8)	(54,14)	**(52,16)**	**(52,14)**	**(54,8)**	**(46,16)**	**(40,20)**	**(36,20)**	(34,16)*
106	24	(30,12)	(48,12)	(48,16)	(50,16)	(54,12)	(50,18)	(48,20)	**(48,18)**	**(50,12)**	**(42,20)**	**(36,24)**	**(32,24)**	(30,20)*
108	26	(28,12)	(46,12)	(46,16)	(48,16)	(52,12)	(48,18)	(46,20)	(46,18)	**(48,12)**	**(40,20)**	**(34,24)**	**(30,24)**	(28,20)*
110	28	(28,8)	(46,8)	(46,12)	(48,12)	(52,8)	(48,14)	(46,16)	(46,14)	(48,8)	**(40,16)**	**(34,20)**	**(30,20)**	(28,16)*
112	30	(30,0)	(48,0)	(48,4)	(50,4)	(54,0)	(50,6)	(48,8)	(48,6)	(50,0)	**(42,8)**	**(36,12)**	**(32,12)**	(30,8)**
114	32	(20,10)	(38,10)	(38,14)	(40,14)	(44,10)	(40,16)	(38,18)	(38,16)	(40,10)	(32,18)	**(26,22)**	**(22,22)**	(20,18)**
116	34	(12,16)	(30,6)	(30,10)	(32,10)	(36,6)	(32,12)	(30,14)	(30,12)	(32,6)	(24,14)	(18,28)∗	**(14,28)**	(12,24) ∗ ∗
118	36	(6,18)	(24,18)	(24,22)	(26,22)	(30,18)	(26,24)	(24,16)	(24,24)	(26,18)	(18,26)	(12,30)	(8,30)∗	(6,26) ∗ ∗
120	38	(2,16)	(20,16)	(20,20)	(22,20)	(26,16)	(22,22)	(20,24)	(20,22)	(22,16)	(14,24)	(8,28)	(4,28)∗	(2,24) ∗ ∗

^{238}U – STRONG ENERGY SPLITTING



λ_0	μ_0	σ_E	σ_B	σ_T	$\frac{B(E2;2_1^+\to 0_1^+)_{\text{th}}}{B(E2;2_1^+\to 0_1^+)_{\text{ex}}}$
14	2	160.370	192.230	352.600	0.80
16	2	161.826	147.258	309.084	<u>1.00</u>
18	2	<u>155.761</u>	<u>118.766</u>	**280.135**	1.22
20	2	161.369	130.944	286.705	1.46
14	4	182.535	157.965	340.500	<u>1.00</u>
16	4	163.762	127.718	291.480	1.22
18	4	162.192	132.754	294.946	1.46
20	4	182.105	177.395	359.500	1.73
14	6	249.246	128.980	378.226	1.22
16	6	248.081	131.491	379.572	1.46
18	6	251.268	174.330	425.598	1.73
20	6	251.314	243.373	494.687	2.01

Table 7. Energy and B(E2) RMS deviations σ_E (in keV) and σ_B (in W.u.) and their sum σ_T (in relative units, r.u.) calculated for various (λ, μ)-values in ^{238}U after the completion of Step (ii') ref to the modified algorithm. The lowest σ_E - and σ_B - values are underlined and that for σ_T is bolded.

$(\lambda_0, \mu_0) = (16, 2)$

$(\lambda_f, \mu_f) = (86, 6)$

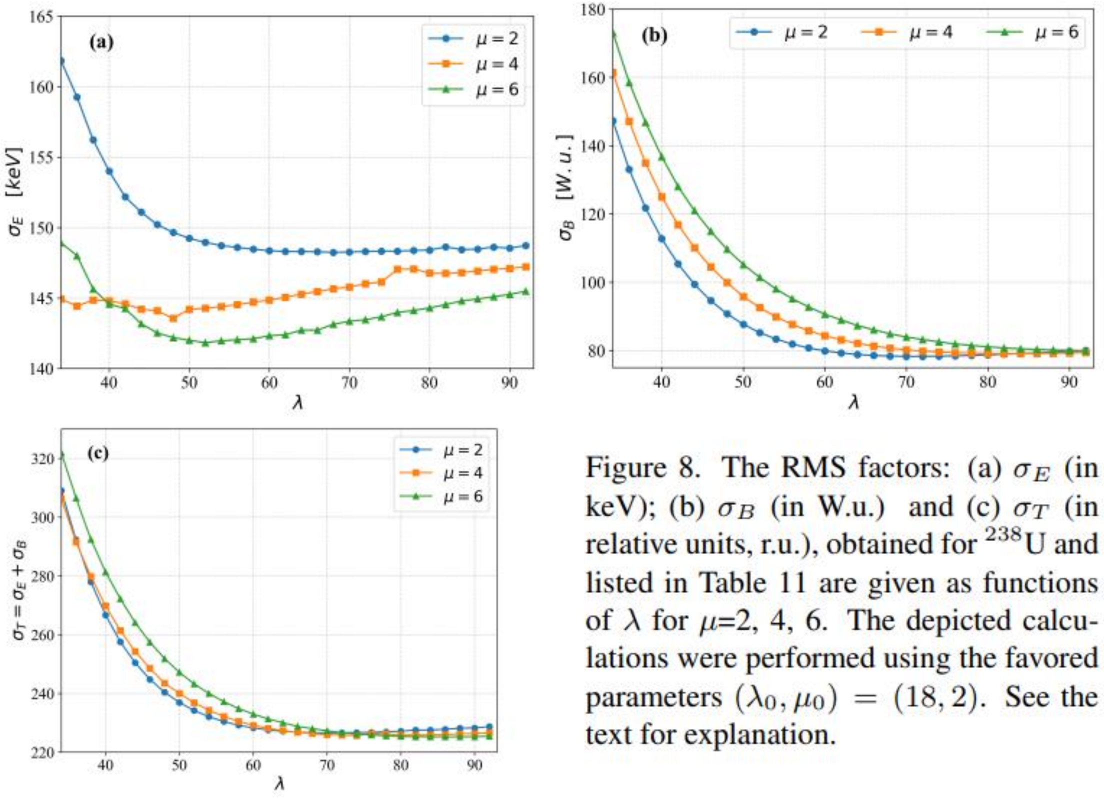
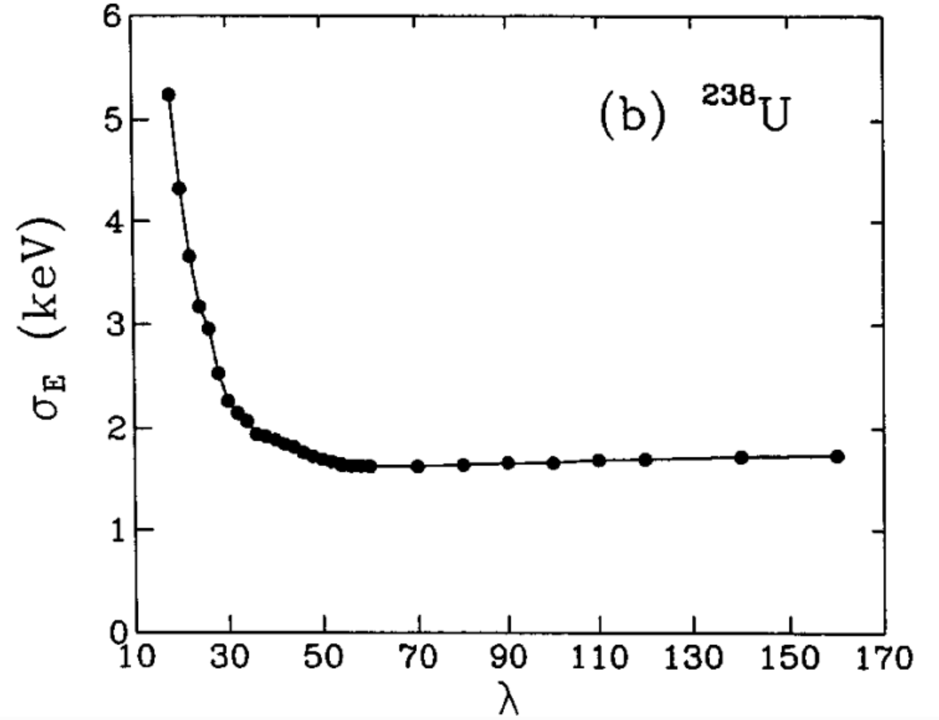


Figure 8. The RMS factors: (a) σ_E (in keV); (b) σ_B (in W.u.) and (c) σ_T (in relative units, r.u.), obtained for ^{238}U and listed in Table 11 are given as functions of λ for μ=2, 4, 6. The depicted calculations were performed using the favored parameters $(\lambda_0, \mu_0) = (18, 2)$. See the text for explanation.

COMPARISON WITH PAST RESEARCH – ^{238}U



μ=2

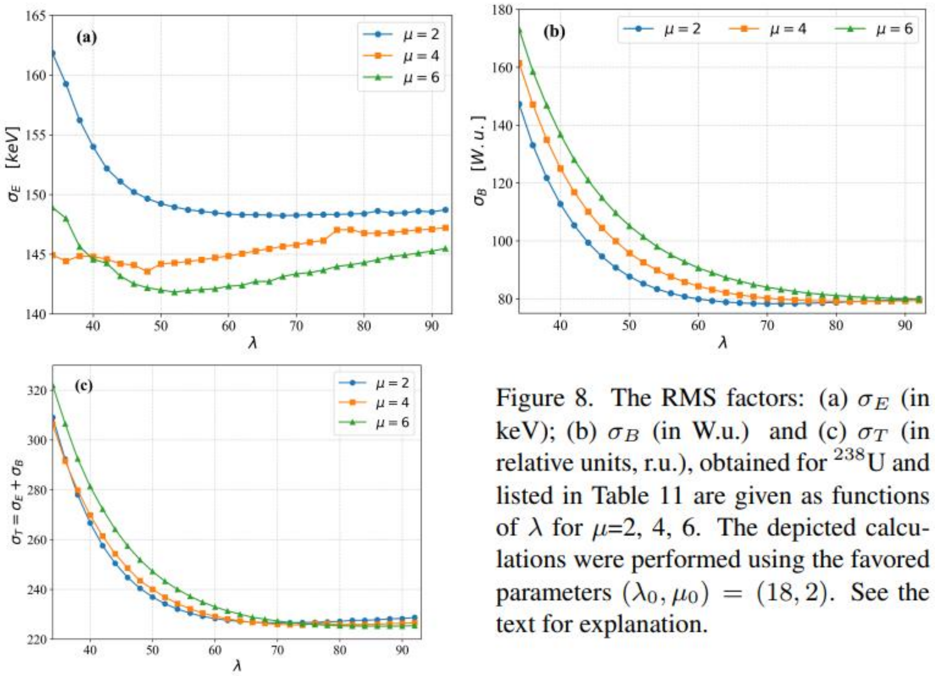


Figure 8. The RMS factors: (a) σ_E (in keV); (b) σ_B (in W.u.) and (c) σ_T (in relative units, r.u.), obtained for ^{238}U and listed in Table 11 are given as functions of λ for μ=2, 4, 6. The depicted calculations were performed using the favored parameters $(\lambda_0, \mu_0) = (18, 2)$. See the text for explanation.

- PRC97: (λ, μ) = (54-60, 2)

Current results: (λ_f, μ_f) = (86, 6)

COMPARISON WITH PROXY SU(3) – ^{238}U

Favored irreps ^{238}U:

Current results:

(λ_f, μ_f) =(86, 6)

λ/μ= 14.3

Proxy SU(3):

(λ, μ) =(90, 4)

λ/μ= 22.5

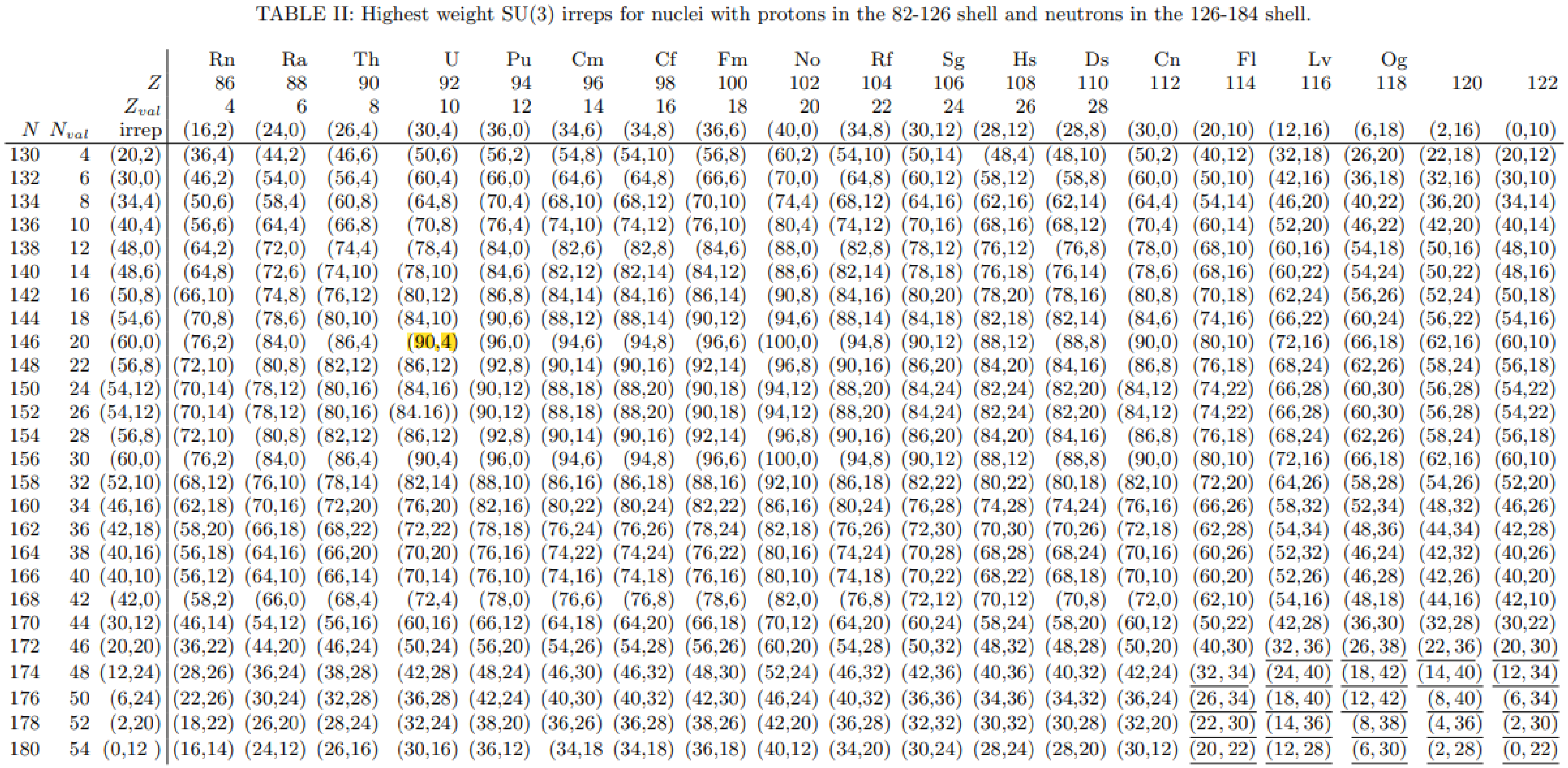
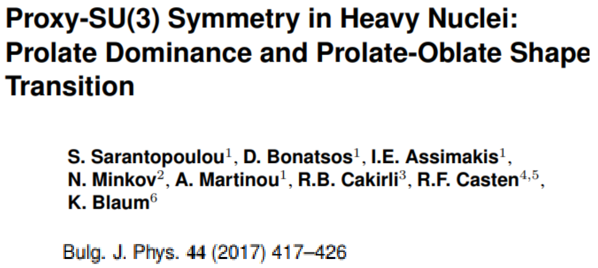


TABLE II: Highest weight SU(3) irreps for nuclei with protons in the 82-126 shell and neutrons in the 126-184 shell.

			Rn	Ra	Th	U	Pu	Cm	Cf	Fm	No	Rf	Sg	Hs	Ds	Cn	Fl	Lv	Og		
		Z	86	88	90	92	94	96	98	100	102	104	106	108	110	112	114	116	118	120	122
		Z_{val}	4	6	8	10	12	14	16	18	20	22	24	26	28						
N	N_{val}	irrep	(16,2)	(24,0)	(26,4)	(30,4)	(36,0)	(34,6)	(34,8)	(36,6)	(40,0)	(34,8)	(30,12)	(28,12)	(28,8)	(30,0)	(20,10)	(12,16)	(6,18)	(2,16)	(0,10)
130	4	(20,2)	(36,4)	(44,2)	(46,6)	(50,6)	(56,2)	(54,8)	(54,10)	(56,8)	(60,2)	(54,10)	(50,14)	(48,4)	(48,10)	(50,2)	(40,12)	(32,18)	(26,20)	(22,18)	(20,12)
132	6	(30,0)	(46,2)	(54,0)	(56,4)	(60,4)	(66,0)	(64,6)	(64,8)	(66,6)	(70,0)	(64,8)	(60,12)	(58,12)	(58,8)	(60,0)	(50,10)	(42,16)	(36,18)	(32,16)	(30,10)
134	8	(34,4)	(50,6)	(58,4)	(60,8)	(64,8)	(70,4)	(68,10)	(68,12)	(70,10)	(74,4)	(68,12)	(64,16)	(62,16)	(62,14)	(64,4)	(54,14)	(46,20)	(40,22)	(36,20)	(34,14)
136	10	(40,4)	(56,6)	(64,4)	(66,8)	(70,8)	(76,4)	(74,10)	(74,12)	(76,10)	(80,4)	(74,12)	(70,16)	(68,16)	(68,12)	(70,4)	(60,14)	(52,20)	(46,22)	(42,20)	(40,14)
138	12	(48,0)	(64,2)	(72,0)	(74,4)	(78,4)	(84,0)	(82,6)	(82,8)	(84,6)	(88,0)	(82,8)	(78,12)	(76,12)	(76,8)	(78,0)	(68,10)	(60,16)	(54,18)	(50,16)	(48,10)
140	14	(48,6)	(64,8)	(72,6)	(74,10)	(78,10)	(84,6)	(82,12)	(82,14)	(84,12)	(88,6)	(82,14)	(78,18)	(76,18)	(76,14)	(78,6)	(68,16)	(60,22)	(54,24)	(50,22)	(48,16)
142	16	(50,8)	(66,10)	(74,8)	(76,12)	(80,12)	(86,8)	(84,14)	(84,16)	(86,14)	(90,8)	(84,16)	(80,20)	(78,20)	(78,16)	(80,8)	(70,18)	(62,24)	(56,26)	(52,24)	(50,18)
144	18	(54,6)	(70,8)	(78,6)	(80,10)	(84,10)	(90,6)	(88,12)	(88,14)	(90,12)	(94,6)	(88,14)	(84,18)	(82,18)	(82,14)	(84,6)	(74,16)	(66,22)	(60,24)	(56,22)	(54,16)
146	20	(60,0)	(76,2)	(84,0)	(86,4)	(90,4)	(96,0)	(94,6)	(94,8)	(96,6)	(100,0)	(94,8)	(90,12)	(88,12)	(88,8)	(90,0)	(80,10)	(72,16)	(66,18)	(62,16)	(60,10)
148	22	(56,8)	(72,10)	(80,8)	(82,12)	(86,12)	(92,8)	(90,14)	(90,16)	(92,14)	(96,8)	(90,16)	(86,20)	(84,20)	(84,16)	(86,8)	(76,18)	(68,24)	(62,26)	(58,24)	(56,18)
150	24	(54,12)	(70,14)	(78,12)	(80,16)	(84,16)	(90,12)	(88,18)	(88,20)	(90,18)	(94,12)	(88,20)	(84,24)	(82,24)	(82,20)	(84,12)	(74,22)	(66,28)	(60,30)	(56,28)	(54,22)
152	26	(54,12)	(70,14)	(78,12)	(80,16)	(84.16))	(90,12)	(88,18)	(88,20)	(90,18)	(94,12)	(88,20)	(84,24)	(82,24)	(82,20)	(84,12)	(74,22)	(66,28)	(60,30)	(56,28)	(54,22)
154	28	(56,8)	(72,10)	(80,8)	(82,12)	(86,12)	(92,8)	(90,14)	(90,16)	(92,14)	(96,8)	(90,16)	(86,20)	(84,20)	(84,16)	(86,8)	(76,18)	(68,24)	(62,26)	(58,24)	(56,18)
156	30	(60,0)	(76,2)	(84,0)	(86,4)	(90,4)	(96,0)	(94,6)	(94,8)	(96,6)	(100,0)	(94,8)	(90,12)	(88,12)	(88,8)	(90,0)	(80,10)	(72,16)	(66,18)	(62,16)	(60,10)
158	32	(52,10)	(68,12)	(76,10)	(78,14)	(82,14)	(88,10)	(86,16)	(86,18)	(88,16)	(92,10)	(86,18)	(82,22)	(80,22)	(80,18)	(82,10)	(72,20)	(64,26)	(58,28)	(54,26)	(52,20)
160	34	(46,16)	(62,18)	(70,16)	(72,20)	(76,20)	(82,16)	(80,22)	(80,24)	(82,22)	(86,16)	(80,24)	(76,28)	(74,28)	(74,24)	(76,16)	(66,26)	(58,32)	(52,34)	(48,32)	(46,26)
162	36	(42,18)	(58,20)	(66,18)	(68,22)	(72,22)	(78,18)	(76,24)	(76,26)	(78,24)	(82,18)	(76,26)	(72,30)	(70,30)	(70,26)	(72,18)	(62,28)	(54,34)	(48,36)	(44,34)	(42,28)
164	38	(40,16)	(56,18)	(64,16)	(66,20)	(70,20)	(76,16)	(74,22)	(74,24)	(76,22)	(80,16)	(74,24)	(70,28)	(68,28)	(68,24)	(70,16)	(60,26)	(52,32)	(46,24)	(42,32)	(40,26)
166	40	(40,10)	(56,12)	(64,10)	(66,14)	(70,14)	(76,10)	(74,16)	(74,18)	(76,16)	(80,10)	(74,18)	(70,22)	(68,22)	(68,18)	(70,10)	(60,20)	(52,26)	(46,28)	(42,26)	(40,20)
168	42	(42,0)	(58,2)	(66,0)	(68,4)	(72,4)	(78,0)	(76,6)	(76,8)	(78,6)	(82,0)	(76,8)	(72,12)	(70,12)	(70,8)	(72,0)	(62,10)	(54,16)	(48,18)	(44,16)	(42,10)
170	44	(30,12)	(46,14)	(54,12)	(56,16)	(60,16)	(66,12)	(64,18)	(64,20)	(66,18)	(70,12)	(64,20)	(60,24)	(58,24)	(58,20)	(60,12)	(50,22)	(42,28)	(36,30)	(32,28)	(30,22)
172	46	(20,20)	(36,22)	(44,20)	(46,24)	(50,24)	(56,20)	(54,26)	(54,28)	(56,26)	(60,20)	(54,28)	(50,32)	(48,32)	(48,28)	(50,20)	(40,30)	(32, 36)	(26, 38)	(22, 36)	(20, 30)
174	48	(12,24)	(28,26)	(36,24)	(38,28)	(42,28)	(48,24)	(46,30)	(46,32)	(48,30)	(52,24)	(46,32)	(42,36)	(40,36)	(40,32)	(42,24)	(32, 34)	(24, 40)	(18, 42)	(14, 40)	(12, 34)
176	50	(6,24)	(22,26)	(30,24)	(32,28)	(36,28)	(42,24)	(40,30)	(40,32)	(42,30)	(46,24)	(40,32)	(36,36)	(34,36)	(34,32)	(36,24)	(26, 34)	(18, 40)	(12, 42)	(8, 40)	(6, 34)
178	52	(2,20)	(18,22)	(26,20)	(28,24)	(32,24)	(38,20)	(36,26)	(36,28)	(38,26)	(42,20)	(36,28)	(32,32)	(30,32)	(30,28)	(32,20)	(22, 30)	(14, 36)	(8, 38)	(4, 36)	(2, 30)
180	54	(0,12)	(16,14)	(24,12)	(26,16)	(30,16)	(36,12)	(34,18	(34,18)	(36,18)	(40,12)	(34,20)	(30,24)	(28,24)	(28,20)	(30,12)	(20, 22)	(12, 28)	(6, 30)	(2, 28)	(0, 22)



Proxy-SU(3) Symmetry in Heavy Nuclei: Prolate Dominance and Prolate-Oblate Shape Transition

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Bulg. J. Phys. 44 (2017) 417–426

SUMMARY - COMPARISON WITH PRC97

- Quantitative differences and qualitative similarities.
- Same characteristic behavior of the curves.
- Same range for the values of the Hamiltonian

parameters for ^{168}Er.

We see a definitive consistency in the description despite the significant differences in the approach now vs. PRC97.

SUMMARY - COMPARISON WITH PROXY SU(3)

SU(3) VBM with effective charges:

^{166}Er: $(\lambda_f, \mu_f) = (20, 4)$

^{168}Er: $(\lambda_f, \mu_f) = (22, 8)$

^{178}Hf: $(\lambda_f, \mu_f) = (46, 20)$

^{238}U: $(\lambda_f, \mu_f) = (86, 6)$

Proxy SU(3):

^{166}Er: $(\lambda, \mu) = (52, 14)$

^{168}Er: $(\lambda, \mu) = (54, 12)$

^{178}Hf: $(\lambda, \mu) = (42, 20)$

^{238}U: $(\lambda, \mu) = (90, 4)$

The results are very similar for the nuclei with intermediate and strong energy splitting. Significant deviations between the two models are observed for nuclei with weak energy splitting.

DISCUSSION

- Independent validity check without effective charges
  - Suggested algorithms with B(E2) normalization
- Preliminary results
- Open questions
  - Alternative way of determination of the best description? Alternative criteria, other than minimal σ_T
  - Further tests with algorithms with B(E2) normalization

THANK YOU FOR YOUR ATTENTION!

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