Vector-boson-model study of SU(3) symmetry in heavy deformed nuclei

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Contents

The vector-bosons as elementary nuclear collective excitations [P. Raychev, R. Roussev, Sov. J. Nucl. Phys. 27, 792 (1978)]

Assumption:
Certain class of nuclear collective properties can be described by two types of elementary excitations created by the operators
$\boldsymbol{\xi}^{+}$, $\boldsymbol{\eta}^{+}$
→ vectors defined in Fock space

$$\xi_\nu = (-1)^\nu \partial/\partial\xi^{+}_{-\nu} \; ; \quad \eta_\nu = (-1)^\nu \partial/\partial\eta^{+}_{-\nu} \; , \qquad \nu = 1, 0, -1$$

$$\xi^\mu = (-1)^\mu \xi_{-\mu} \; ; \quad \eta^\mu = (-1)^\mu \eta_{-\mu}$$

→ closing boson commutation relations

$$[\xi^\mu, \xi^{+}_\nu] = [\eta^\mu, \eta^{+}_\nu] = \delta_{\mu\nu} \; , \qquad \mu, \nu = 1, 0, -1$$

Vector-boson realization of SU(3) algebra

Angular momentum

$$L_m = -\sqrt{2}\sum_{\mu,\nu} C^{1m}_{1\mu 1\nu}(\xi^+_\mu \xi_\nu + \eta^+_\mu \eta_\nu) \ , \qquad m = 0, \pm 1$$

Quadrupole momentum

$$Q_k = \sqrt{6}\sum_{\mu,\nu} C^{2k}_{1\mu 1\nu}(\xi^+_\mu \xi_\nu + \eta^+_\mu \eta_\nu) \ , \qquad k = 0, \pm 1, \pm 2$$

$\rightarrow$ closing SU(3) algebra

$$\begin{aligned} [L_m, L_n] &= -\sqrt{2} C^{1m+n}_{1m1n} L_{m+n} \\ [L_m, Q_n] &= \sqrt{6} C^{2m+n}_{1m2n} Q_{m+n} \\ [Q_m, Q_n] &= 3\sqrt{10} C^{1m+n}_{2m2n} L_{m+n} \end{aligned}$$

Vector boson model (VBM) Hamiltonian and basis

Rotation invariants reducing $SU(3)$ to $O(3)$:

$L^2 = \sum_m (-1)^m L_m L_{-m}$

$L \cdot Q \cdot L = \sum_{M,m,m'} (-1)^M C^{2M}_{1m1m'} Q_{-M} L_m L_{m'}$

$A^+ A, \qquad A^+ = (\boldsymbol{\xi}^+)^2 (\boldsymbol{\eta}^+)^2 - (\boldsymbol{\xi}^+ \cdot \boldsymbol{\eta}^+)^2$

Vector-boson Hamiltonian with broken SU(3) symmetry

$H_{VBM} = g_1 L^2 + g_2 L \cdot Q \cdot L + g_3 A^+ A$

$SU(3) \supset O(3) \supset O(2)$

Basis [V. Bargmann and M. Moshinsky, Nucl. Phys. **23**, 177 (1961)]

$$\left| \begin{array}{c} (\lambda, \mu) \\ \alpha, L, M \end{array} \right\rangle = P^{(\lambda,\mu,\alpha,L,M)}(\xi^+_\nu, \eta^+_\nu)|0\rangle, \quad \alpha \; - \; O(3) \text{ multiplicity q.n.}$$

Diagonalization and spectrum [N.M. et al, PRC 55 2345 (1997)]

$$\left| \begin{array}{c} (\lambda,\mu) \\ \omega_i^L, L, L \end{array} \right\rangle = \sum_{j=1}^{d_L} C_{i,j}^L \left| \begin{array}{c} (\lambda,\mu) \\ \alpha_j, L, L \end{array} \right\rangle$$

λ, μ even, $\lambda > \mu$: $\alpha = 0, 1, 2, \ldots \mu/2 = \alpha_{max}$

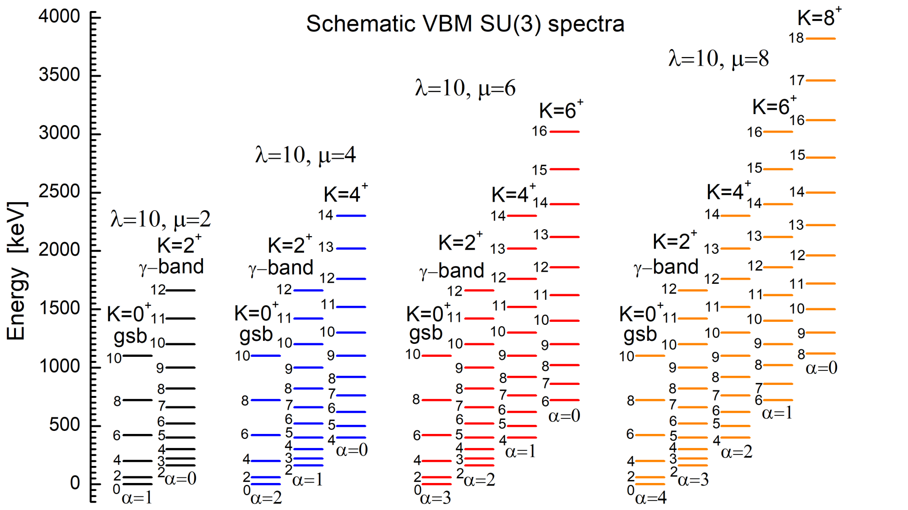
$(\lambda, \mu) \rightarrow \{(\alpha_i, L_{\alpha_i})\} \rightarrow SU(3)$ multiplet

$K = \mu - 2\alpha, \quad N = \lambda + 2\mu$ – number of vector bosons

Energy bands

$\alpha_{max} = \mu/2$: $L = 0^+, 2^+, 4^+, \ldots, L_{max} = \lambda$ gsb
$\alpha_{max} - 1$: $L = 2^+, 3^+, 4^+, \ldots, L_{max} = \lambda + 2$ γ band
$\alpha_{max} - 2$: $L = 4^+, 5^+, 6^+, \ldots, L_{max} = \lambda + 4$ $K^\pi = 4^+$ band; ...
$\alpha = 0$: $L = \mu, \mu + 1, \mu + 2, \ldots, L_{max} = \lambda + \mu$ $K = \mu$ band

VBM spectra for different SU(3) irreps: SU(3) multiplets



B(E2) transition rates

$$B(E2;\omega_i^L \to \omega_{i'}^{L+k}) = \frac{1}{2L+1}\begin{pmatrix} L+k & 2 & L \\ -L & 0 & L \end{pmatrix}^{-2} \times \left| \left\langle \begin{matrix} (\lambda,\mu) \\ \omega_{i'}^{L+k}, L+k, L \end{matrix} \right| Q_0 \left| \begin{matrix} (\lambda,\mu) \\ \omega_i^L, L, L \end{matrix} \right\rangle \right|^2$$

$$R_1(L) = \frac{B(E2;L_\gamma \to L_g)}{B(E2;L_\gamma \to (L-2)_g)}, \quad \text{L even}$$

$$R_2(L) = \frac{B(E2;L_\gamma \to (L+2)_g)}{B(E2;L_\gamma \to L_g)}, \quad \text{L even}$$

$$R_3(L) = \frac{B(E2;L_\gamma \to (L+1)_g)}{B(E2;L_\gamma \to (L-1)_g)}, \quad \text{L odd}$$

$$R_4(L) = \frac{B(E2;L_g \to (L-2)_g)}{B(E2;(L-2)_g \to (L-4)_g)}, \quad \text{L odd}$$

VBM model fit for given (λ, μ)-irrep [N.M. et al, PRC 55 2345 (1997)]

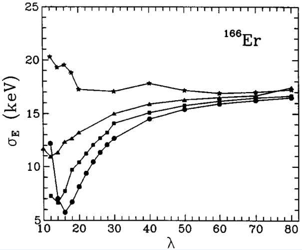
$\rightarrow$ **Ground- and γ-band levels and B(E2) transition ratios $R_i(L)$ ($i = 1, ..., 4$) taken up to $L = 10$**

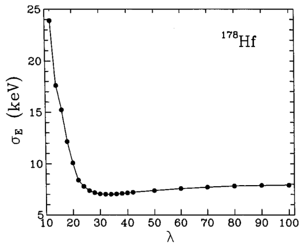
$\Rightarrow$ Combined RMS deviation factor

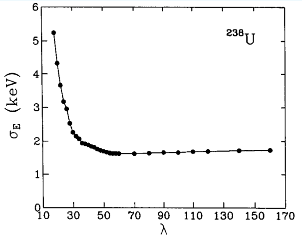
$$\sigma_E = \sqrt{\frac{1}{n_E} \sum_{\nu=1}^{n_E} \left[E_\nu^{\text{th}} - E_\nu^{\text{exp}}\right]^2}, \quad \sigma_{B(R)} = \sqrt{\frac{1}{n_B} \sum_{\nu=1}^{n_B} \left[R_\nu^{\text{th}} - R_\nu^{\text{exp}}\right]^2}$$

$\sigma_T = \sigma_E + w \cdot \sigma_{B(R)} \rightarrow$ total RMS deviation minimized
$w \rightarrow$ weight factor

Favored SU(3) irreps [N. M. et al, PRC 55 2345 (1997)]







The (λ, μ) values are varied to get best model description of energy levels and transition ratios

• $\mu = 2$, ■ $\mu = 4$, ▲ $\mu = 6$, ★ $\mu = 8$

Favored SU(3) multiplets [PRC 55 2345 (1997)]

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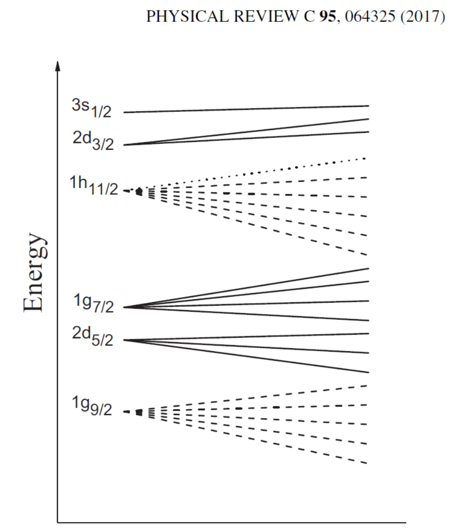
TABLE II. The parameters of the fits of the energy levels and the transition ratios [Eqs. (20) and (21)] of the nuclei investigated are listed for the (λ,μ) multiplets which provide the best model descriptions. The Hamiltonian parameters g_1, g_2, and g_3 [Eq. (5)] are given in keV. The quantities σ_E (in keV) and σ_B (dimensionless) represent the energy [Eq. (44)] and the transition [Eq. (45)] rms factors, respectively. The splitting ratios ΔE_2 [Eq. (46), dimensionless] and the vector-boson numbers N [Eq. (9)] are also given.

Nucl	ΔE_2	λ,μ	σ_E	σ_B	g_1	g_2	g_3	N
^{164}Dy	9.4	16,2	14.1	0.52	−1.159	−0.321	−0.590	20
^{164}Er	8.4	18,2	8.1	0.14	3.625	−0.238	−0.513	22
^{166}Er	8.8	16,2	5.8	0.47	2.942	−0.235	−0.572	20
^{168}Er	9.3	20,2	3.2	0.28	4.000	−0.181	−0.401	24
^{168}Yb	10.2	20,2	7.9	0.27	0.500	−0.271	−0.501	24
^{172}Yb	17.6	⩾80,2	6.8	0.12	9.875	−0.017	−0.052	84
^{176}Hf	14.2	⩾70,2	15.0	0.17	9.547	−0.030	−0.062	74
^{178}Hf	11.6	34,2	7.0	0.86	8.322	−0.083	−0.213	38
^{238}U	22.6	⩾60,2	1.6	0.08	−37.697	−0.360	−0.098	64

$$\Delta E_2 = (E_{2_\gamma} - E_{2_g})/E_{2_g}$$

Proxy SU(3) mapping (D. Bonatsos et al)



Here the (λ, μ) values are determined through the valence numbers in the mapped Nilsson orbitals

Proxy SU(3) irreps (D. Bonatsos et al)

ANALYTIC PREDICTIONS FOR NUCLEAR SHAPES, . . .   PHYSICAL REVIEW C **95**, 064326 (2017)

TABLE II. Most leading SU(3) irreps [34,35] for nuclei with protons in the 50-82 shell and neutrons in the 82-126 shell. Boldface numbers indicate nuclei with $R_{4/2} = E(4_1^+)/E(2_1^+) \geqslant 2.8$, while * denotes nuclei with $2.8 > R_{4/2} \geqslant 2.5$, and ** labels a few nuclei with $R_{4/2}$ ratios slightly below 2.5, shown for comparison, while no irreps are shown for any other nuclei with $R_{4/2} < 2.5$. For the rest of the nuclei shown (using normal fonts and without any special signs attached) the $R_{4/2}$ ratios are still unknown [46]. Irreps corresponding to oblate shapes are underlined.

N	N_{val}	Z	Ba	Ce	Nd	Sm	Gd	Dy	Er	Yb	Hf	W	Os	Pt
		Z_{val}	56	58	60	62	64	66	68	70	72	74	76	78
		irrep	6	8	10	12	14	16	18	20	22	24	26	28
			(18,0)	(18,4)	(20,4)	(24,0)	(20,6)	(18,8)	(18,6)	(20,0)	(12,8)	(6,12)	(2,12)	(0,8)
88	6	(24,0)	(42,0)*	(42,4)*	(44,4)*									
90	8	(26,4)	**(44,4)**	**(44,8)**	**(46,8)**	**(50,4)**	**(46,10)**	**(44,12)**	(44,10)*	(46,4)*	(38,12)*			
92	10	(30,4)	**(48,4)**	**(48,8)**	**(50,8)**	**(54,4)**	**(50,10)**	**(48,12)**	**(48,10)**	**(50,4)**	(42,12)*			
94	12	(36,0)	(54,0)	**(54,4)**	**(56,4)**	**(60,0)**	**(56,6)**	**(54,8)**	**(54,6)**	**(56,0)**	**(48,8)**	**(42,12)**	(38,12)*	
96	14	(34,6)	(52,6)	(52,10)	**(54,10)**	**(58,6)**	**(54,12)**	**(52,14)**	**(52,12)**	**(54,6)**	**(46,14)**	**(40,18)**	(36,18)*	
98	16	(34,8)	(52,8)	(52,12)	(54,12)	**(58,8)**	**(54,14)**	**(52,16)**	**(52,14)**	**(54,8)**	**(46,16)**	**(40,20)**	(36,20)*	
100	18	(36,6)	(54,6)	(54,10)	(56,10)	(60,6)	**(56,12)**	**(54,14)**	**(54,12)**	**(56,6)**	**(48,14)**	**(42,18)**	**(38,18)**	(36,14)*
102	20	(40,0)	(58,0)	(58,4)	(60,4)	(64,0)	**(60,6)**	**(58,8)**	**(58,6)**	**(60,0)**	**(52,8)**	**(46,12)**	**(42,12)**	(40,8)*
104	22	(34,8)	(52,8)	(52,12)	(54,12)	(58,8)	(54,14)	**(52,16)**	**(52,14)**	**(54,8)**	**(46,16)**	**(40,20)**	**(36,20)**	(34,16)*
106	24	(30,12)	(48,12)	(48,16)	(50,16)	(54,12)	(50,18)	(48,20)	**(48,18)**	**(50,12)**	**(42,20)**	**(36,24)**	**(32,24)**	(30,20)*
108	26	(28,12)	(46,12)	(46,16)	(48,16)	(52,12)	(48,18)	(46,20)	(46,18)	**(48,12)**	**(40,20)**	**(34,24)**	**(30,24)**	(28,20)*
110	28	(28,8)	(46,8)	(46,12)	(48,12)	(52,8)	(48,14)	(46,16)	(46,14)	(48,8)	**(40,16)**	**(34,20)**	**(30,20)**	(28,16)*
112	30	(30,0)	(48,0)	(48,4)	(50,4)	(54,0)	(50,6)	(48,8)	(48,6)	(50,0)	**(42,8)**	**(36,12)**	**(32,12)**	(30,8)**
114	32	(20,10)	(38,10)	(38,14)	(40,14)	(44,10)	(40,16)	(38,18)	(38,16)	(40,10)	(32,18)	**(26,22)**	**(22,22)**	(20,18)**
116	34	(12,16)	(30,6)	(30,10)	(32,10)	(36,6)	(32,12)	(30,14)	(30,12)	(32,6)	(24,14)	(18,28)*	**(14,28)**	(12,24)**
118	36	(6,18)	(24,18)	(24,22)	(26,22)	(30,18)	(26,24)	(24,16)	(24,24)	(26,18)	(18,26)	(12,30)	(8,30)*	(6,26)**
120	38	(2,16)	(20,16)	(20,20)	(22,20)	(26,16)	(22,22)	(20,24)	(20,22)	(22,16)	(14,24)	(8,28)	(4,28)*	(2,24)**

Proxy SU(3) irreps [S. Sarantopoulou, D. Bonatsos et al, BJP 44, 417 (2017)]

TABLE II: Highest weight SU(3) irreps for nuclei with protons in the 82-126 shell and neutrons in the 126-184 shell.

			Rn	Ra	Th	U	Pu	Cm	Cf	Fm	No	Rf	Sg	Hs	Ds	Cn	Fl	Lv	Og		
		Z	86	88	90	92	94	96	98	100	102	104	106	108	110	112	114	116	118	120	122
		Z_{val}	4	6	8	10	12	14	16	18	20	22	24	26	28						
N	N_{val}	irrep	(16,2)	(24,0)	(26,4)	(30,4)	(36,0)	(34,6)	(34,8)	(36,6)	(40,0)	(34,8)	(30,12)	(28,12)	(28,8)	(30,0)	(20,10)	(12,16)	(6,18)	(2,16)	(0,10)
130	4	(20,2)	(36,4)	(44,2)	(46,6)	(50,6)	(56,2)	(54,8)	(54,10)	(56,8)	(60,2)	(54,10)	(50,14)	(48,4)	(48,10)	(50,2)	(40,12)	(32,18)	(26,20)	(22,18)	(20,12)
132	6	(30,0)	(46,2)	(54,0)	(56,4)	(60,4)	(66,0)	(64,6)	(64,8)	(66,6)	(70,0)	(64,8)	(60,12)	(58,12)	(58,8)	(60,0)	(50,10)	(42,16)	(36,18)	(32,16)	(30,10)
134	8	(34,4)	(50,6)	(58,4)	(60,8)	(64,8)	(70,4)	(68,10)	(68,12)	(70,10)	(74,4)	(68,12)	(64,16)	(62,16)	(62,14)	(64,4)	(54,14)	(46,20)	(40,22)	(36,20)	(34,14)
136	10	(40,4)	(56,6)	(64,4)	(66,8)	(70,8)	(76,4)	(74,10)	(74,12)	(76,10)	(80,4)	(74,12)	(70,16)	(68,16)	(68,12)	(70,4)	(60,14)	(52,20)	(46,22)	(42,20)	(40,14)
138	12	(48,0)	(64,2)	(72,0)	(74,4)	(78,4)	(84,0)	(82,6)	(82,8)	(84,6)	(88,0)	(82,8)	(78,12)	(76,12)	(76,8)	(78,0)	(68,10)	(60,16)	(54,18)	(50,16)	(48,10)
140	14	(48,6)	(64,8)	(72,6)	(74,10)	(78,10)	(84,6)	(82,12)	(82,14)	(84,12)	(88,6)	(82,14)	(78,18)	(76,18)	(76,14)	(78,6)	(68,16)	(60,22)	(54,24)	(50,22)	(48,16)
142	16	(50,8)	(66,10)	(74,8)	(76,12)	(80,12)	(86,8)	(84,14)	(84,16)	(86,14)	(90,8)	(84,16)	(80,20)	(78,20)	(78,16)	(80,8)	(70,18)	(62,24)	(56,26)	(52,24)	(50,18)
144	18	(54,6)	(70,8)	(78,6)	(80,10)	(84,10)	(90,6)	(88,12)	(88,14)	(90,12)	(94,6)	(88,14)	(84,18)	(82,18)	(82,14)	(84,6)	(74,16)	(66,22)	(60,24)	(56,22)	(54,16)
146	20	(60,0)	(76,2)	(84,0)	(86,4)	(90,4)	(96,0)	(94,6)	(94,8)	(96,6)	(100,0)	(94,8)	(90,12)	(88,12)	(88,8)	(90,0)	(80,10)	(72,16)	(66,18)	(62,16)	(60,10)
148	22	(56,8)	(72,10)	(80,8)	(82,12)	(86,12)	(92,8)	(90,14)	(90,16)	(92,14)	(96,8)	(90,16)	(86,20)	(84,20)	(84,16)	(86,8)	(76,18)	(68,24)	(62,26)	(58,24)	(56,18)
150	24	(54,12)	(70,14)	(78,12)	(80,16)	(84,16)	(90,12)	(88,18)	(88,20)	(90,18)	(94,12)	(88,20)	(84,24)	(82,24)	(82,20)	(84,12)	(74,22)	(66,28)	(60,30)	(56,28)	(54,22)
152	26	(54,12)	(70,14)	(78,12)	(80,16)	(84.16))	(90,12)	(88,18)	(88,20)	(90,18)	(94,12)	(88,20)	(84,24)	(82,24)	(82,20)	(84,12)	(74,22)	(66,28)	(60,30)	(56,28)	(54,22)
154	28	(56,8)	(72,10)	(80,8)	(82,12)	(86,12)	(92,8)	(90,14)	(90,16)	(92,14)	(96,8)	(90,16)	(86,20)	(84,20)	(84,16)	(86,8)	(76,18)	(68,24)	(62,26)	(58,24)	(56,18)
156	30	(60,0)	(76,2)	(84,0)	(86,4)	(90,4)	(96,0)	(94,6)	(94,8)	(96,6)	(100,0)	(94,8)	(90,12)	(88,12)	(88,8)	(90,0)	(80,10)	(72,16)	(66,18)	(62,16)	(60,10)
158	32	(52,10)	(68,12)	(76,10)	(78,14)	(82,14)	(88,10)	(86,16)	(86,18)	(88,16)	(92,10)	(86,18)	(82,22)	(80,22)	(80,18)	(82,10)	(72,20)	(64,26)	(58,28)	(54,26)	(52,20)
160	34	(46,16)	(62,18)	(70,16)	(72,20)	(76,20)	(82,16)	(80,22)	(80,24)	(82,22)	(86,16)	(80,24)	(76,28)	(74,28)	(74,24)	(76,16)	(66,26)	(58,32)	(52,34)	(48,32)	(46,26)
162	36	(42,18)	(58,20)	(66,18)	(68,22)	(72,22)	(78,18)	(76,24)	(76,26)	(78,24)	(82,18)	(76,26)	(72,30)	(70,30)	(70,26)	(72,18)	(62,28)	(54,34)	(48,36)	(44,34)	(42,28)
164	38	(40,16)	(56,18)	(64,16)	(66,20)	(70,20)	(76,16)	(74,22)	(74,24)	(76,22)	(80,16)	(74,24)	(70,28)	(68,28)	(68,24)	(70,16)	(60,26)	(52,32)	(46,24)	(42,32)	(40,26)
166	40	(40,10)	(56,12)	(64,10)	(66,14)	(70,14)	(76,10)	(74,16)	(74,18)	(76,16)	(80,10)	(74,18)	(70,22)	(68,22)	(68,18)	(70,10)	(60,20)	(52,26)	(46,28)	(42,26)	(40,20)
168	42	(42,0)	(58,2)	(66,0)	(68,4)	(72,4)	(78,0)	(76,6)	(76,8)	(78,6)	(82,0)	(76,8)	(72,12)	(70,12)	(70,8)	(72,0)	(62,10)	(54,16)	(48,18)	(44,16)	(42,10)
170	44	(30,12)	(46,14)	(54,12)	(56,16)	(60,16)	(66,12)	(64,18)	(64,20)	(66,18)	(70,12)	(64,20)	(60,24)	(58,24)	(58,20)	(60,12)	(50,22)	(42,28)	(36,30)	(32,28)	(30,22)
172	46	(20,20)	(36,22)	(44,20)	(46,24)	(50,24)	(56,20)	(54,26)	(54,28)	(56,26)	(60,20)	(54,28)	(50,32)	(48,32)	(48,28)	(50,20)	(40,30)	(32, 36)	(26, 38)	(22, 36)	(20, 30)
174	48	(12,24)	(28,26)	(36,24)	(38,28)	(42,28)	(48,24)	(46,30)	(46,32)	(48,30)	(52,24)	(46,32)	(42,36)	(40,36)	(40,32)	(42,24)	(32, 34)	(24, 40)	(18, 42)	(14, 40)	(12, 34)
176	50	(6,24)	(22,26)	(30,24)	(32,28)	(36,28)	(42,24)	(40,30)	(40,32)	(42,30)	(46,24)	(40,32)	(36,36)	(34,36)	(34,32)	(36,24)	(26, 34)	(18, 40)	(12, 42)	(8, 40)	(6, 34)
178	52	(2,20)	(18,22)	(26,20)	(28,24)	(32,24)	(38,20)	(36,26)	(36,28)	(38,26)	(42,20)	(36,28)	(32,32)	(30,32)	(30,28)	(32,20)	(22, 30)	(14, 36)	(8, 38)	(4, 36)	(2, 30)
180	54	(0,12)	(16,14)	(24,12)	(26,16)	(30,16)	(36,12)	(34,18	(34,18)	(36,18)	(40,12)	(34,20)	(30,24)	(28,24)	(28,20)	(30,12)	(20, 22)	(12, 28)	(6, 30)	(2, 28)	(0, 22)

Favored VBM and proxy SU(3) irreps

Nucl	ΔE_2	λ, μ	σ_E	σ_B	g_1	g_2	g_3	N
^{164}Dy	9.4	16, 2	14.1	0.52	−1.159	−0.321	−0.590	20
	9.4	52, 16	19.8	0.46	−18.558	−0.194	−0.052	84
^{164}Er	8.4	18, 2	8.1	0.14	3.625	−0.238	−0.513	22
	8.4	52, 12	18.5	0.15	−8.805	−0.158	−0.059	76
^{166}Er	8.8	16, 2	5.8	0.47	2.942	−0.235	−0.572	20
	8.8	52, 14	19.1	0.43	−11.081	−0.235	−0.153	80
^{168}Er	9.3	20, 2	3.2	0.28	4.000	−0.181	−0.401	24
	9.3	54, 12	12.8	0.21	−7.799	−0.136	−0.053	78
^{168}Yb	10.2	20, 2	7.9	0.27	0.500	−0.271	−0.501	24
	10.2	54, 8	10.9	0.24	−6.536	−0.151	−0.071	70
^{172}Yb	17.6	$\geq$ 80, 2	6.8	0.12	9.875	−0.017	−0.052	84
	17.6	60, 2	7.4	0.12	9.531	−0.024	−0.091	64
^{176}Hf	14.2	$\geq$ 70, 2	15.0	0.17	9.547	−0.030	−0.062	74
	14.2	46, 16	15.5	0.16	−28.637	−0.262	−0.106	78
^{178}Hf	11.6	34, 2	7.0	0.86	8.322	−0.083	−0.213	38
	11.6	42, 20	7.6	0.86	−43.408	−0.354	−0.102	82
^{238}U	22.6	$\geq$ 60, 2	1.7	0.08	−38.112	−0.363	−0.098	64
	22.6	90, 4	1.7	0.08	−32.992	−0.215	−0.042	98

Need for unambiguous determination of the favored (λ,μ)-irreps

⇒ Direct comparison of B(E2)-values with data (not their ratios)

⇒ Taking into account both energy and B(E2) description quality to select the favored (λ,μ)- irrep

⇒ Combined RMS deviation factor

$$\sigma_E = \sqrt{\frac{1}{n_E}\sum_{\nu=1}^{n_E}\left[E_\nu^{\text{th}} - E_\nu^{\text{exp}}\right]^2},\ \sigma_B^w = \sqrt{\frac{1}{n_B}\sum_{\nu=1}^{n_B} w_\nu \left[B(E2)_\nu^{\text{th}} - B(E2)_\nu^{\text{exp}}\right]^2}$$

$$\sigma_B = \sqrt{\frac{1}{n_B}\sum_{\nu=1}^{n_B}\left[B(E2)_\nu^{\text{th}} - B(E2)_\nu^{\text{exp}}\right]^2}$$

$\sigma_T^w = \sigma_E + \sigma_B^w \rightarrow$ minimized for given (λ,μ)- irrep
$\sigma_T = \sigma_E + \sigma_B \rightarrow$ total RMS factor to determine the favored (λ,μ)- irrep

(λ, μ)-dependence of the B(E2) transition values

Table 1. Energy and B(E2) RMS deviations σ_E (in keV) and σ_B (in W.u.) obtained for various (λ, μ)-irreps in ^{166}Er without use of scaling factors in the B(E2)-values

λ	μ	σ_E	σ_B	λ	μ	σ_E	σ_B	λ	μ	σ_E	σ_B
16	2	57.674	38.782	30	2	60.954	593.529	52	2	65.796	2179.215
16	4	76.169	50.112	30	4	62.772	651.024	52	4	66.375	2273.434
16	6	70.519	81.042	30	6	65.755	712.053	52	6	67.312	2370.952
16	8	169.096	127.712	30	8	70.548	777.065	52	8	68.613	2471.913
16	10	375.453	185.438	30	10	78.117	846.748	52	10	70.361	2576.495
16	12	538.432	245.178	30	12	95.193	922.229	52	12	73.116	2684.930
16	14	692.485	309.237	30	14	132.573	1005.555	52	14	76.093	2797.531

Table 2. Energy and B(E2) RMS deviations σ_E (in keV) and σ_B (in W.u.) obtained for various (λ, μ)-irreps in ^{166}Er by using the scaling factors (2) and (3) with (λ_0, μ_0)=(16,2)

λ	μ	σ_E	σ_B	λ	μ	σ_E	σ_B	λ	μ	σ_E	σ_B
16	2	57.674	38.782	30	2	60.961	48.820	52	2	65.753	59.249
16	4	58.118	37.261	30	4	62.965	39.902	52	4	66.467	51.927
16	6	70.178	40.899	30	6	66.252	34.888	52	6	67.524	45.993
16	8	108.108	46.092	30	8	71.280	33.383	52	8	69.080	41.417
16	10	192.247	51.738	30	10	78.487	34.484	52	10	70.905	38.165
16	12	168.264	57.356	30	12	88.227	37.125	52	12	73.370	36.167
16	14	185.742	62.792	30	14	100.609	40.494	52	14	76.444	35.292

Introduction of irrep-dependent effective charges

$$e^2_{\text{eff-intra}} = \left(\frac{\lambda_0 + \mu_0 + 1}{\lambda + \mu + 1}\right)^2 \quad \text{for intraband transitions}$$

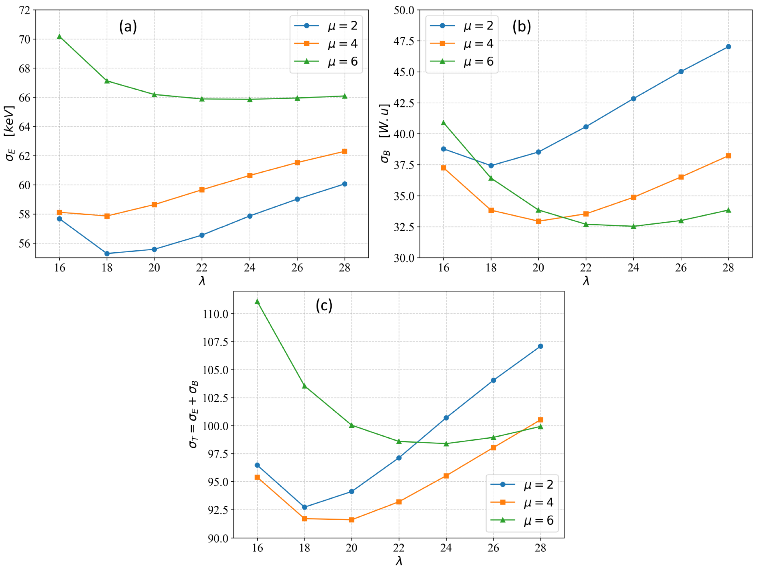
$$e^2_{\text{eff-inter}} = \left(\frac{\mu_0 + 1}{\mu + 1}\right)^2 \quad \text{for interband transitions}$$

It can be shown that [using a connection $(\lambda, \mu) \to (\beta, \gamma)$]:

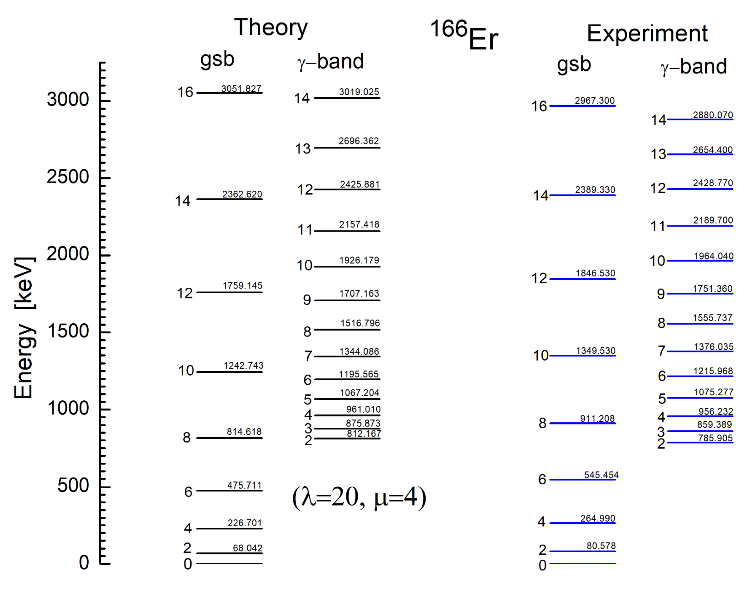
$$e^2_{\text{eff-intra}} \approx \left(\frac{\beta_0}{\beta}\right)^2, \quad e^2_{\text{eff-inter}} \approx \left(\frac{\gamma_0}{\gamma}\right)^2$$

Explanations on the application by Dimana

Calculation with irrep-dependent effective charges in ^{166} Er



VBM spectrum for ^{166}Er in the favoured (20,4)-irrep

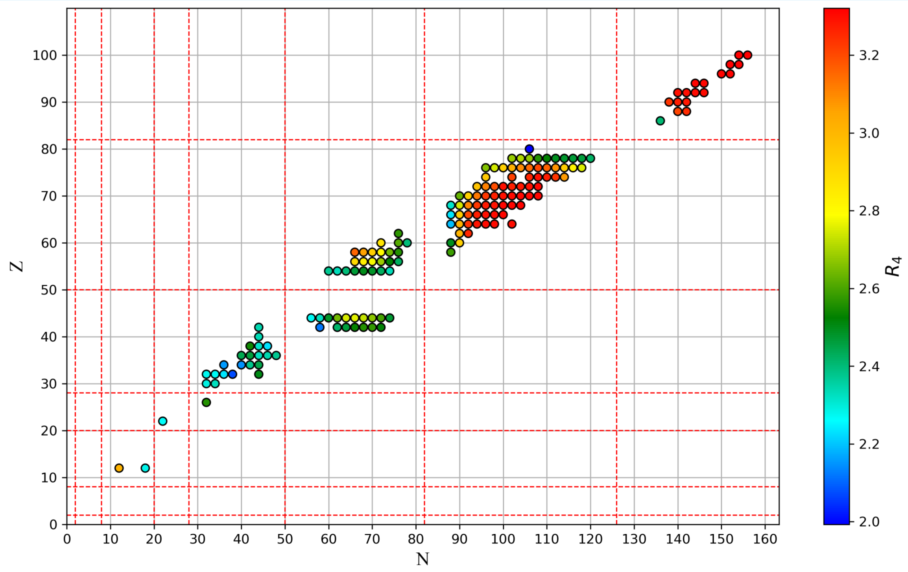


VBM B(E2)s for ^{166}Er in the favoured (20,4)-irrep

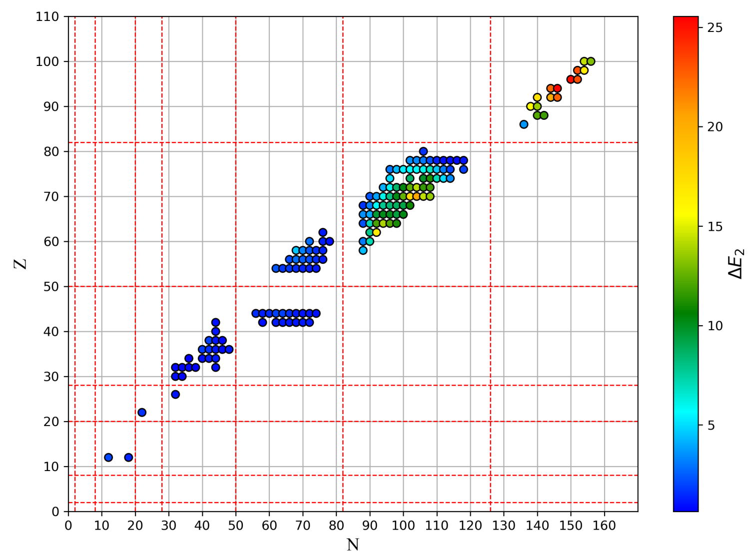
L_{b_i}	L_{b_f}	$B(E2)_{\text{th}}$	$B(E2)_{\text{exp}}$	$B(E2)_{\text{th}}/B(E2)_{\text{exp}}$
2g	0g	254.613	217(5)	1.17
4g	2g	358.137	312(11)	1.15
6g	4g	383.41	370(20)	1.04
8g	6g	384.777	373(14)	1.03
10g	8g	373.154	390(17)	0.96
12g	10g	352.329	372(21)	0.95
2γ	0g	4.892	5.17(21)	0.95
2γ	2g	8.299	9.6(6)	0.86
2γ	4g	0.608	0.78(4)	0.78
4γ	2g	2.285	1.98(12)	1.15
4γ	6g	1.593	2.1(14)	0.76
6γ	4g	1.438	0.88(6)	1.63
6γ	8g	2.486	1.9(3)	1.31
8γ	6g	0.954	0.52(5)	1.83
8γ	10g	3.412	1.5(0)	2.27
4γ	2γ	148.027	138(9)	1.07
4γ	3γ	335.804	370(3)	0.91
5γ	3γ	234.85	300(4)	0.78
5γ	4γ	242.793	310(4)	0.78
6γ	4γ	280.643	225(16)	1.25
8γ	6γ	318.545	250(23)	1.27
9γ	7γ	331.105	370(15)	0.89

PERSPECTIVE

$R_4 = E(4_1^+)/E(2_1^+)$ ratio in the nuclei with presence of γ band



Ground-γ band splitting factor $\Delta E_2 = (E_{2_\gamma} - E_{2_g})/E_{2_g}$



Concluding remarks

- The VBM algorithm is applicable with the proxy SU(3) defined irreps. Favored VBM irreps: considerably differ by the proxy irreps for nuclei with small ΔE_2 and appear close to them in nuclei with large ground-γ band mutual displacement.
- The proxy SU(3) symmetry acquires further physical significance related to the structure of nuclear collective excited spectra (ground and γ bands, in particular).
- Common VBM and proxy SU(3) symmetry framework: Detailed analysis and systematics of spectra and transition rates in wide ranges of even-even deformed nuclei.

SU(3) irreducible representations (irreps) and Casimir operators

$U(n) \supset U(3) \supset SU(3)$

$U(3)$ representation $[f_1, f_2, f_3]$, $\quad (f_1 \geq f_2 \geq f_3)$

$SU(3)$ irreps: (λ, μ), $\quad \lambda = f_1 - f_2$, $\mu = f_2 - f_3$

$SU(3)$ **invariants (Casimir operators):**

$\hat{C}_2 \equiv \sum_i \hat{F}_i \hat{F}_i \simeq \frac{1}{4} \hat{Q} \cdot \hat{Q} + \frac{3}{4} \hat{L}^2$

$\hat{C}_3 \equiv \sum_{ijk} \hat{F}_i \hat{F}_j \hat{F}_k$

Eigenvalues (calc. in SU(3) matr. repr., Baird, Biedenharn, JMP 1963)

$\langle \hat{C}_2 \rangle \sim \lambda^2 + \mu^2 + \lambda\mu + 3(\lambda + \mu)$

$\langle \hat{C}_3 \rangle \sim (\lambda - \mu)(\lambda + 2\mu + 3)(2\lambda + \mu + 3)$

$dim(\lambda, \mu) = 1/2(\lambda + 1)(\mu + 1)(\lambda + \mu + 2)$

$\boldsymbol{\xi}^+$, $\boldsymbol{\eta}^+ \rightarrow$ O(3) vectors transforming under two independent $(\lambda, \mu) = (1, 0)$ irreps